Internet Appendix

Overview of the Internet Appendix

Appendix IA.A: Derives the optimal pattern of trade at settlement across different strike prices of the SPX options that a potential manipulator would follow.

Appendix IA.B: Explains the details of how fair quotes of individual SPX options at settlement are calculated to benchmark settlement prices of SPX options.

Appendix IA.C: Explains how different benchmarks for gauging the deviation of the VIX value at settlement are calculated.

Figures IA.1, IA.2, and IA.3: Visualize the relationship between VIX and SPX derivatives, the timeline of the settlement, and the potential process to manipulate VIX.

Figure IA.4: Shows that VIX sensitivity and trade volume of call options drop as they become more OTM.

Figure IA.5: Describes how the ΔK measure in the VIX formula is calculated.

Figure IA.6: Graphs VIX sensitivity and trade volume as a function of moneyness using alternative bins of moneyness.

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Figure IA.8: Shows that other characteristics of put options are similar and change continuously around the ΔK discontinuity cutoff.

Figure IA.9: Equivalent of Figure 3 and IA.8 for call options. It shows the discontinuity in VIX sensitivity and trade volume around the ΔK cutoff for call options.

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Figure IA.12: Shows the trade volume at different prices for EURO STOXX 50 options around the €0.5 cutoff during the VSTOXX settlement days.

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Table IA.2: Reports that VIX sensitivity and the settlement volume have a jump around the ΔK thresholds, while the strike price, moneyness, and the non-settlement volume change smoothly.

Table IA.3: Shows the relationship between the moneyness of put and call options and their liquidity at non-settlement times.

Table IA.4: Compares the trade volume and open interest of SPX options in months with and without expiring VIX derivatives.

Table IA.5: Compares put SPX option prices at settlement and the bid, ask, and mid-quote of the options at the market open immediately after the settlement and at the close the day before settlement.

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Appendix IA.A. What is the Optimal Trade Volume across Different Strike Prices for Manipulation?

Suppose that a trader wants to manipulate VIX settlement by trading SPX options. There is a trade-off here between the cost of trading SPX options and the benefit from VIX movement. A trader with a long position on expiring VIX futures who submits aggressive buy orders on SPX options faces the cost of overpaying on those options, but receives the benefit of an inflated VIX settlement. Given that VIX has different sensitivity to price changes in different SPX options, the manipulator's question is what the optimal allocation of trade volume would be across different strike prices. Is it better to trade merely on far out-of-the money put options to which VIX is most sensitive, or is it better to spread the volume across the entire option chain? It is shown here that it is optimal to trade the entire chain of options, but allocate higher volume to strikes with higher weight, proportional to the sensitivity of VIX to price changes in each strike.

Suppose there is a stream of SPX options indexed by strike price i. A trader can push the settlement value upwards by buying q_i option contracts of each strike price i. He optimizes over q_i, the trade volume of each strike price i, given the cost of overpaying on that option and the weight of that option in VIX settlement. Suppose there is a true price for option i and the trader can push that price by d_i, which is a function of the quantity traded: $d_i = f(q_i)$. The cost of overpaying on option i is therefore $q_i d_i$ or $q_i f(q_i)$.

Now to calculate the benefit of manipulation, suppose that the trader has M expiring contracts in the upper-level VIX market. Also suppose s_i is the sensitivity of VIX settlement to price movements of SPX option i. VIX moves by $s_i d_i$ if the price of SPX option i deviates by d_i. Note that this is the sensitivity measure calculated from the data in Section 3. Therefore, the gains from pushing the price of strike price i by d_i is $M s_i d_i$ in the upper-level market.

To summarize:

q_i: Trade Volume of Option i

s_i: VIX Sensitivity

d_i: Price Deviation in Option i

M: Number of Positions in VIX Derivatives

d_i is a function of q_i ($d_i = f(q_i)$), therefore:

Costs in the lower-level$= q_i d_i = q_i f(q_i)$

Benefit in the upper-level$= M s_i d_i = M s_i f(q_i)$

A manipulator wants to trade optimally across different strike prices, with a limit to the cost of overspending in SPX options denoted as C. The optimal volume for each strike price is the solution to the following profit-maximization problem:

$$Max_{(q_i)} \sum_{i=1}^{N} (Ms_i f(q_i) - q_i f(q_i))$$

s.t.

$$\sum_{i=1}^{N} q_i f(q_i) = C$$

The solution here is the relative volume across different strikes that equalize the marginal profit from trading each strike. Assuming that price deviation has a constant elasticity with respect to the quantity traded (i.e. $q_i f'(q_i)/f(q_i)$ is constant), the solution implies that the optimal trade volume is exactly proportional to the VIX sensitivities.

$$\frac{q_i^*}{q_j^*} = \frac{s_i}{s_j} \text{ for any strike prices } i \text{ and } j$$

Note that the constant elasticity assumption above is a common assumption and includes a wide range of functional forms for price movements such as linearity in quantity traded or a concave form such as $d_i = \sqrt{q_i}$.

Appendix IA.B. Adjusting the Prices of SPX Options at Open and Previous Close for Changes in Market Condition and Time Decay.

This appendix explains the details of how fair quotes of individual SPX options at settlement (used in the analysis of Figure 6 and Tables IA.5 and IA.6) are calculated. We use the quotes and prices of options on the close of the day before and the open immediately after each settlement to benchmark for objective prices of individual SPX options at settlement. The quotes are adjusted for movements in the SPX index, time decay, and overnight changes in volatility using the Black-Scholes delta, theta, and vega. The Black-Scholes Greeks are calculated as follows:

Notations:

S = SPX index price

K = Strike price of SPX options

σ = Volatility[1]

r = Continuously compounded risk-free interest rate

q = Continuously compounded dividend yield of SPX index[2]

t = Time to expiration

$N()$ = Cumulative standard normal distribution

Calculations:

$$CallsDelta = e^{(-qt)} * N(d_1)$$

$$PutsDelta = e^{(-qt)} * (N(d_1) - 1)$$

$$CallsTheta = \frac{1}{365}[-(\frac{S\sigma e^{-qt}}{2\sqrt{t}} * \frac{e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}) - rKe^{-rt}N(d_2) + qSe^{-qt}N(d_1)]$$

$$PutsTheta = \frac{1}{365}[-(\frac{S\sigma e^{-qt}}{2\sqrt{t}} * \frac{e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}) + rKe^{-rt}N(-d_2) - qSe^{-qt}N(-d_1)]$$

$$Vega = \frac{1}{100}[\frac{Se^{-qt}\sqrt{t} * e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}]$$

where $d_1 = \frac{ln(S/K)+t(r-q+\frac{\sigma^2}{2})}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$

[1] The volatility of each option is calculated as the Black-Scholes implied volatility calculated through an iterative search for the σ that equates the Black-Scholes price with market price.

[2] The data on annual dividend yield of SPX index are obtained from Bloomberg data.

After calculating the Greeks for each SPX option, we adjust the quotes of that option at the open and previous close benchmark as the following:

Market Movements

To adjust the option prices for changes in the market index between the benchmark and the settlement time one needs to know the index value at settlement, which is before the market opens. Because perception of the option traders from the index value is not observable at that time, we use changes in the price of deep ITM options that have an absolute delta close to one to compute the market movements between benchmark and settlement time. The adjustment for index changes is done for quotes of individual SPX options both at the open benchmark and the close of the previous day. The quotes of ITM options at settlement are reported in the settlement imbalance reports. At each settlement, the change of the mid-quote from previous close (open) to settlement is divided by the delta of each option that is at least 20% in-the-money. Afterward, the median within each settlement day is used as the market movement from previous close (open) benchmark to the settlement for that day.[3] We then multiply the market movement by the delta of each individual SPX option to find the objective price changes for each individual option from benchmark to settlement. Finally, we add these individual expected price changes to quotes of the options at benchmark to find their objective value at settlement. The imbalance reports start reporting a full range of strikes in May 2010, so the adjustments here are limited to the sample period after May 2010.

Time Decay

We control for time decay by multiplying the time gap between the benchmark and the settlement by the calculated theta of each individual option. We assume that the amount of overnight time decay is half of a full day. The results are robust to alternative estimations such as allowing for a 24/7 time decay overnight. We also apply time decay adjustment to the quotes of options right after open, though the time gap between settlement and open quotes is usually less than a minute.

Volatility

We proxy for overnight changes in volatility using overnight price changes of the second term VIX futures, measured from the previous close to open of the settlement day. The price change is multiplied by the Vega of each SPX option, and then added to the quotes of the options at the previous close.

[3]Price movements are very close across deep ITM options within each settlement, and the results are not sensitive to the cutoff used for the deep ITM options.

Appendix IA.C. Construction of Different Benchmarks for Measuring VIX Deviations at Settlement.

This appendix explains how different benchmarks are used to gauge the deviation of the VIX value at settlement. The following four benchmarks are computed using the mid-quote for the same range of non-zero bid SPX options that are included in the VIX at settlement and right after the open.

1. Open Benchmark

The open benchmark is a VIX index calculated using the mid-quotes of SPX options right after the market opens. The time gap between the settlement and the timestamp when the options series open in the data is on average 25 seconds. The difference between a VIX index calculated from SPX options at settlement prices and the open benchmark ($VIX_{Settle} - VIX_{OpenBench}$) is defined as the deviations from open, where both measures use the same range of SPX options. In five cases that the option series used to calculate the VIX does not open right after the open in our sample, we use the difference between the VIX settlement and the disseminated VIX at open as the deviation. This measure is available for the settlement days from January 2008 to August 2015. Two settlement days in 2008 are excluded because we cannot closely replicate the reported open price using our intraday data and cannot verify whether the deviations at settlement are caused by data error.

2. Previous Close Benchmark

Here the previous close mid-quotes of the same range of SPX options as above are used to calculate a VIX index at previous close, controlling for the time difference from the previous close to the open. Then the index is adjusted by adding the overnight changes of the price of second-term VIX futures from previous close to open. This measure is available for the settlement days from January 2008 to August 2015.

3. ITM Benchmark

This benchmark uses the mid-quote of in-the-money options right at settlement, reported in settlement imbalance reports. First, the put-call parity implied value of each OTM option is calculated using the ITM options with the same strike price. Then a VIX index is calculated using these implied values. This benchmark starts in May 2010, when the imbalance reports start reporting a full range of strike prices.

4. Previous Close Benchmark Using Black-Scholes Adjusted Quotes

This benchmark measures a similar value as benchmark number 2 (Previous Close Benchmark) but using a different method. Due to the time gap between the previous day close and settlement, we use the adjusted mid-quotes of the previous close (constructed as explained in the Appendix IA.B) to calculate the index. Given that we use the imbalance reports to adjust the option prices for movements of the underlying index level, this benchmark starts from May 2010. The adjusted mid-quotes are used in the normal VIX formula to calculate the previous close benchmark, and then the deviations are calculated as the difference between the settlement and this benchmark.

Open benchmark is used throughout the paper as the main reference because it is the most conservative measure.

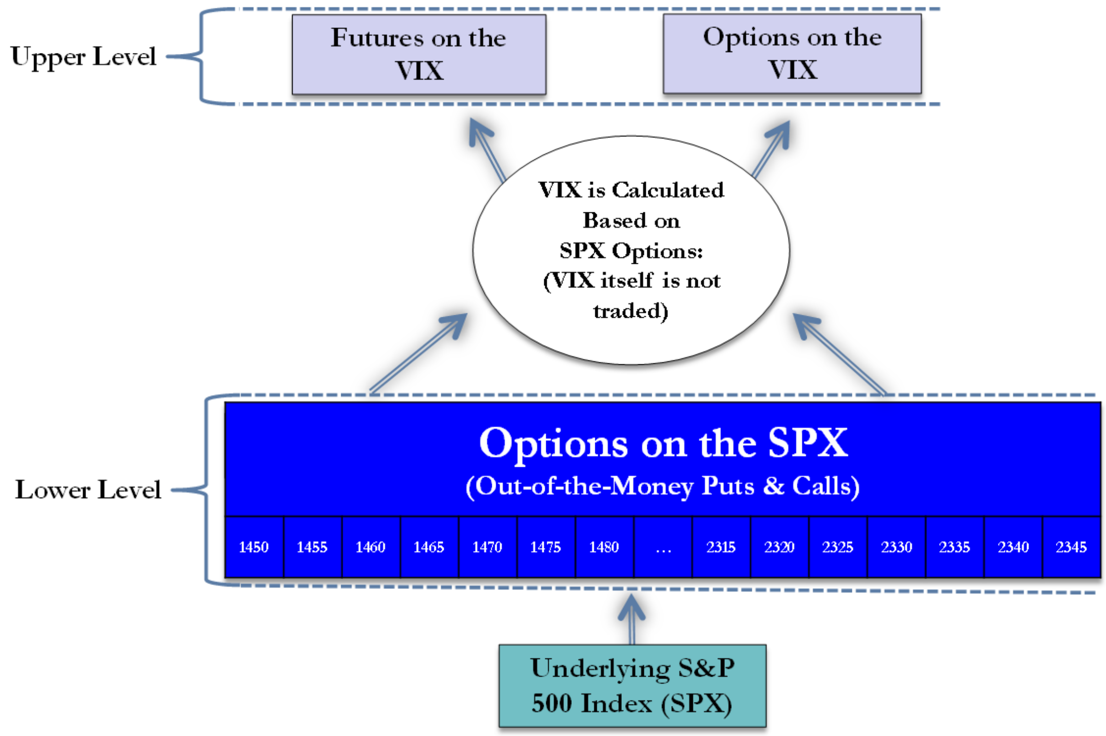


Figure IA.1. SPX and VIX Relationship Description. This diagram shows the relationship between the underlying S&P 500 Index (SPX), options on the SPX, VIX, and derivatives on the VIX. The blue box above shows that specifically out-of-the-money SPX puts and calls are used to calculate the VIX. The white circle above illustrates that the VIX calculation is conducted using the prices of the lower level SPX options. Manipulators trade SPX options in order to influence the VIX settlement value. The two purple boxes at the top show futures and options traded on the VIX. The index itself is not traded directly, but instead buyers and sellers of VIX trade these upper-level VIX derivatives. In order to profit from settlement deviations, manipulators open positions in VIX futures and options prior to settlement and let them settle at maturity.

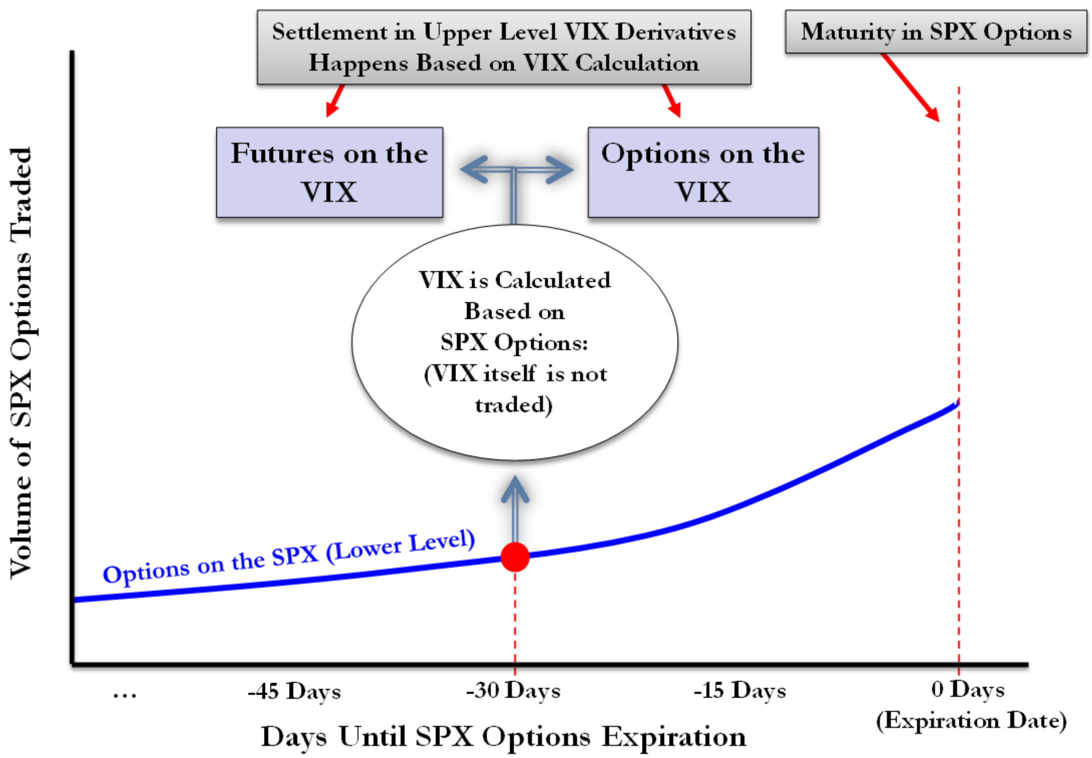


Figure IA.2. Calculation of VIX Settlement. This figure shows the mechanics of the VIX settlement process. The blue line shows the hypothetical trade volume of monthly SPX options by days to expiration. The middle dotted red line shows that VIX settlement is calculated using SPX options with exactly 30 days until expiration. Upper level VIX futures and options (purple boxes) settle based on this VIX settlement value.

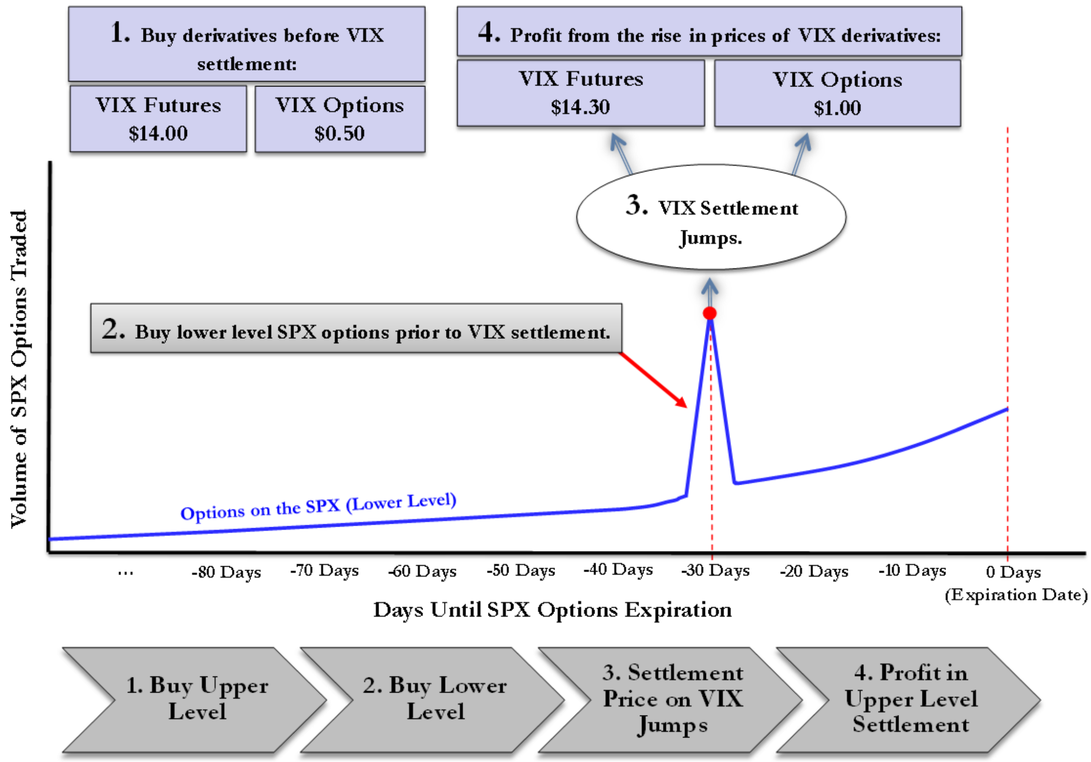
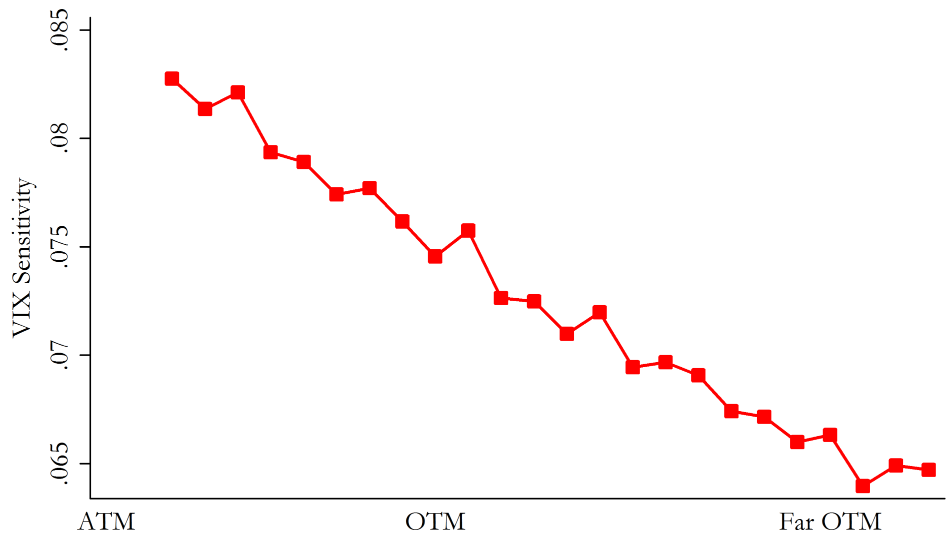


Figure IA.3. Potential Process to Manipulate VIX. This figure shows the most basic steps required to manipulate and profit from VIX settlement deviations. The blue line shows the observed average historical trade volume of monthly SPX options by days to maturity. Manipulators enter into VIX upper level futures and/or options positions some time before VIX settlement occurs. Right before the market opens on settlement day, manipulators submit trade orders for lower level out-of-the-money SPX puts and calls (spike of blue line at 30 days). At market open, trade orders are executed and trade prices are used in VIX settlement calculations. VIX settlement jumps as a result of the increased trading volume and distorts prices of the underlying SPX options. As a result, manipulators will profit from the deviations as their VIX positions converge and settle at calculated VIX settlement value.

Panel A: VIX Sensitivity



Panel B: Settlement Volume

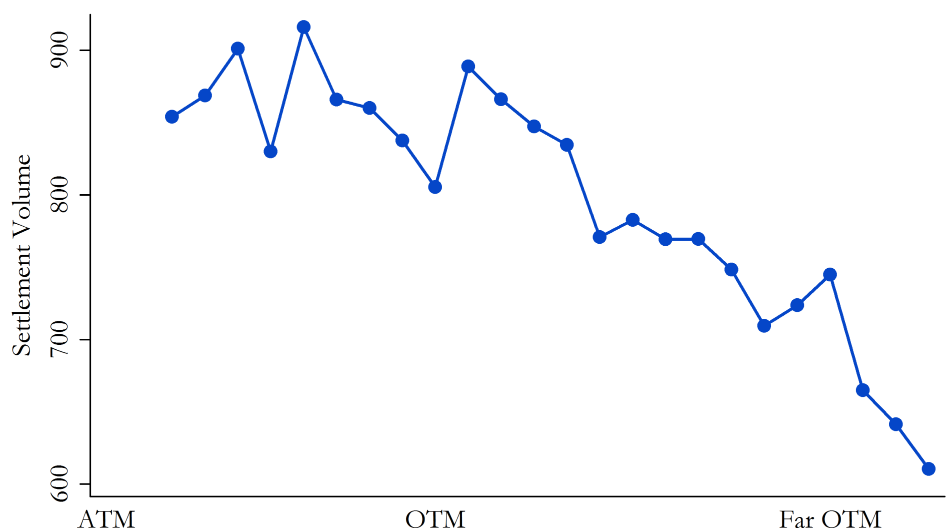


Figure IA.4. VIX Sensitivity and Trade Volume for Call Options with ΔK of Five Units. This figure shows the relationship between trade volume of monthly S&P 500 call options and the sensitivity of VIX settlement value to price movements in those options. The sample consists of call options with ΔK of 5 units used in VIX settlement calculations from January 2008 to April 2015. The options are divided into bins of moneyness, and the VIX sensitivity and trade volume are averaged for each bin, and then over time. Panel A shows the average VIX sensitivity for each bin and Panel B shows the average volume at VIX settlement.

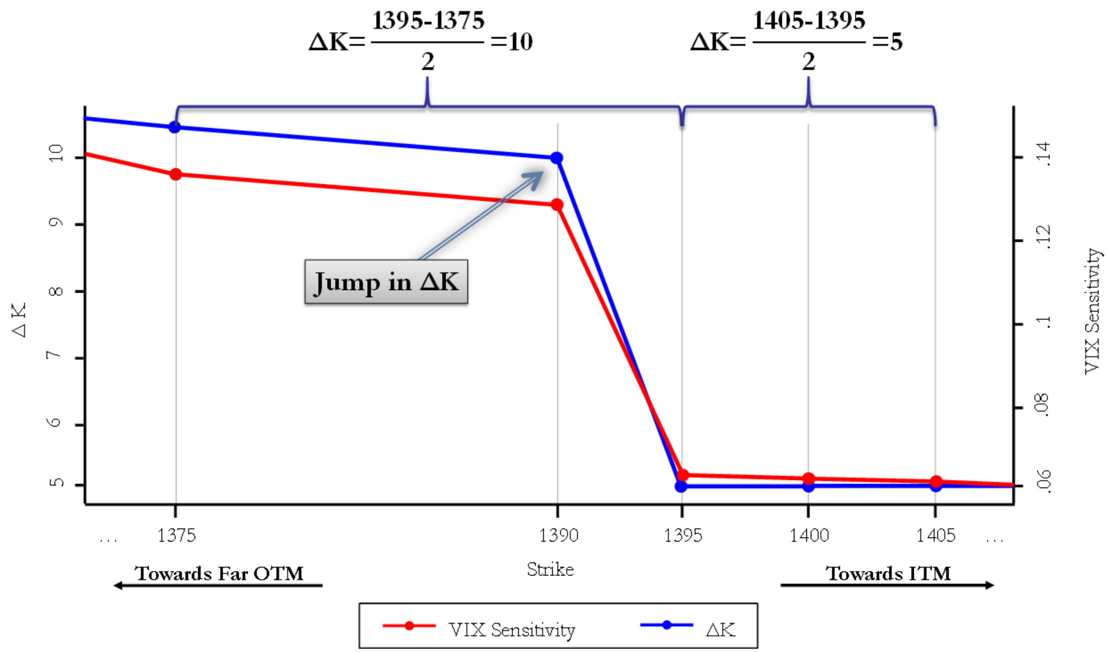


Figure IA.5. ΔK in the VIX Calculation. The graph above illustrates the calculation of ΔK and its direct relationship with VIX sensitivity for a put option. ΔK is calculated for each strike and is defined as the average distance between an option's strike and the nearest strikes above and below that particular option. For strike price 1400, ΔK is five as calculated in the above graph. For strike price 1390, ΔK is 10 as similarly calculated. The ΔK of each option indicates that VIX is approximately twice as sensitive to a \$0.05 price change in options with strike price 1390 compared to options with strike price 1400. The blue line is the ΔK for each strike price and the red line is the sensitivity of VIX to each option as implied by the VIX formula.

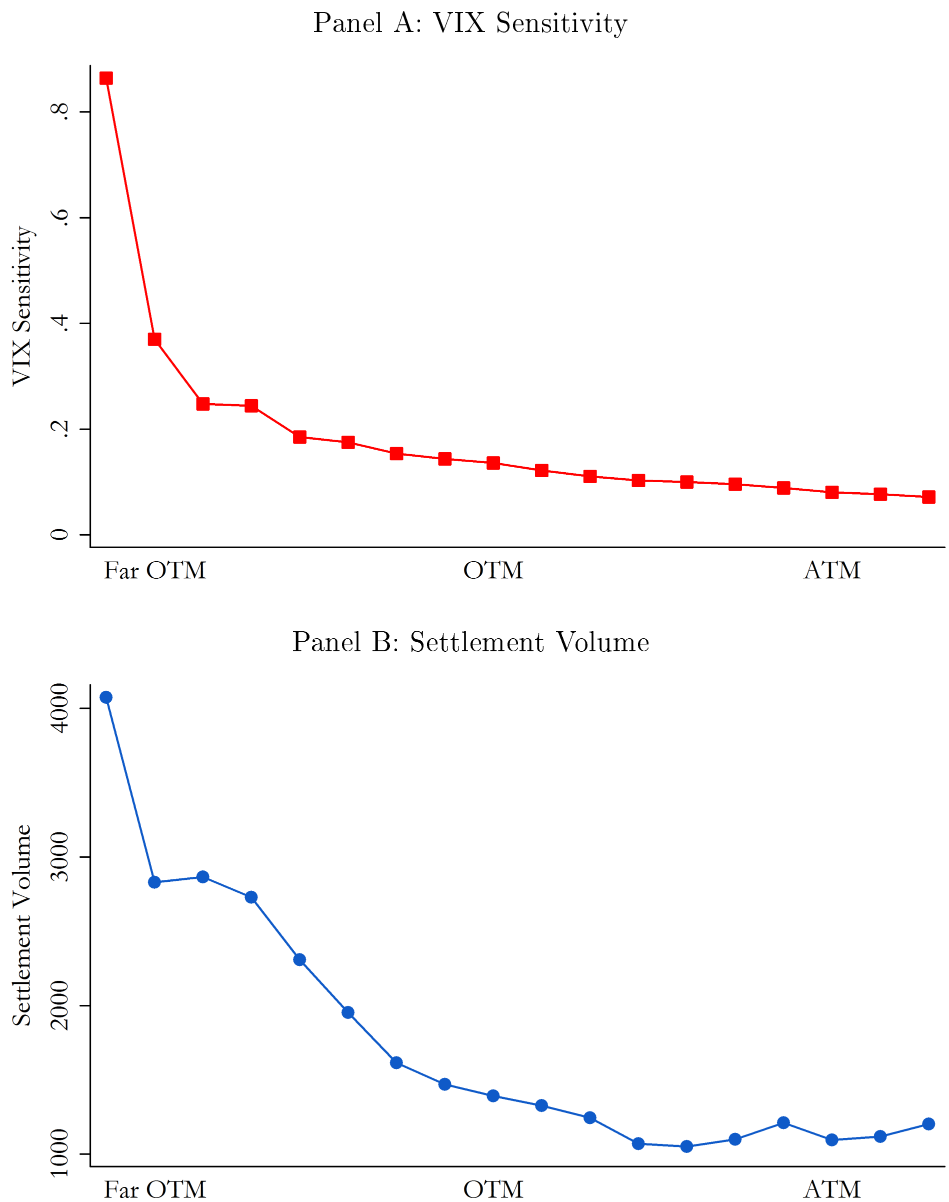
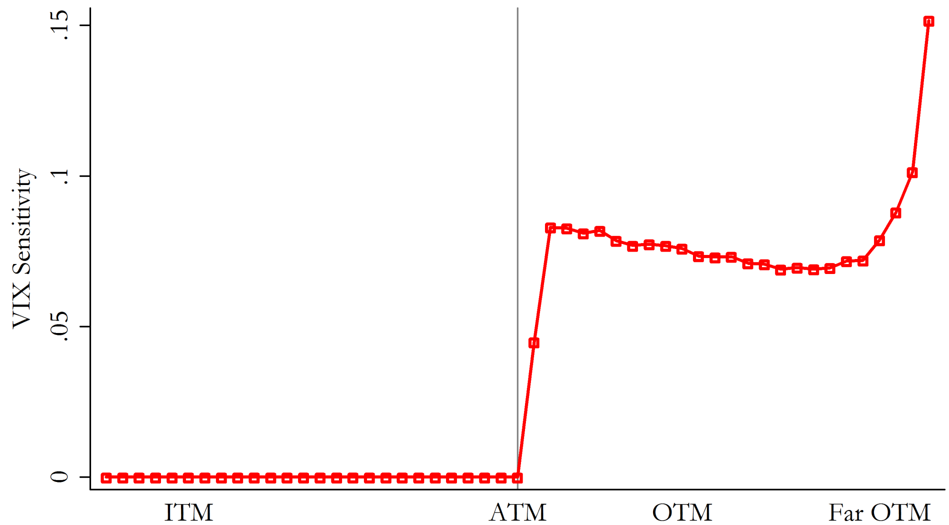


Figure IA.6. VIX Sensitivity and Trade Volume in Bins of Moneyness Standardized by Black-Scholes Implied Volatility. This figure shows the relationship between trade volume of monthly S&P 500 put options and the sensitivity of VIX settlement value to price movements in those options. The sample consists of the options used in VIX settlement calculations from January 2008 to April 2015. VIX sensitivity for each option is calculated as the change in VIX settlement value as a result of five-cent price movements in that option, holding all other prices constant. The moneyness of put options used in settlement calculation are standardized by the Black-Scholes Implied Volatility of ATM options. Afterward, they are divided into bins of moneyness by the standardized moneyness, and the VIX sensitivity and trade volume are averaged for each bin and then over time. Panel A shows the average VIX sensitivity for each bin and Panel B shows the average volume at VIX settlement.

Panel A: Call Options VIX Sensitivity



Panel B: Call Options Trade Volume at Settlement

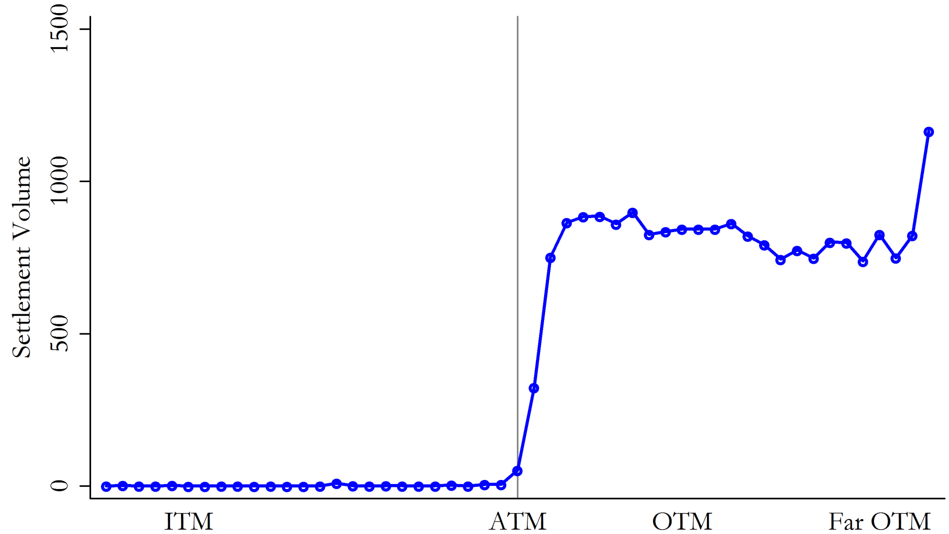
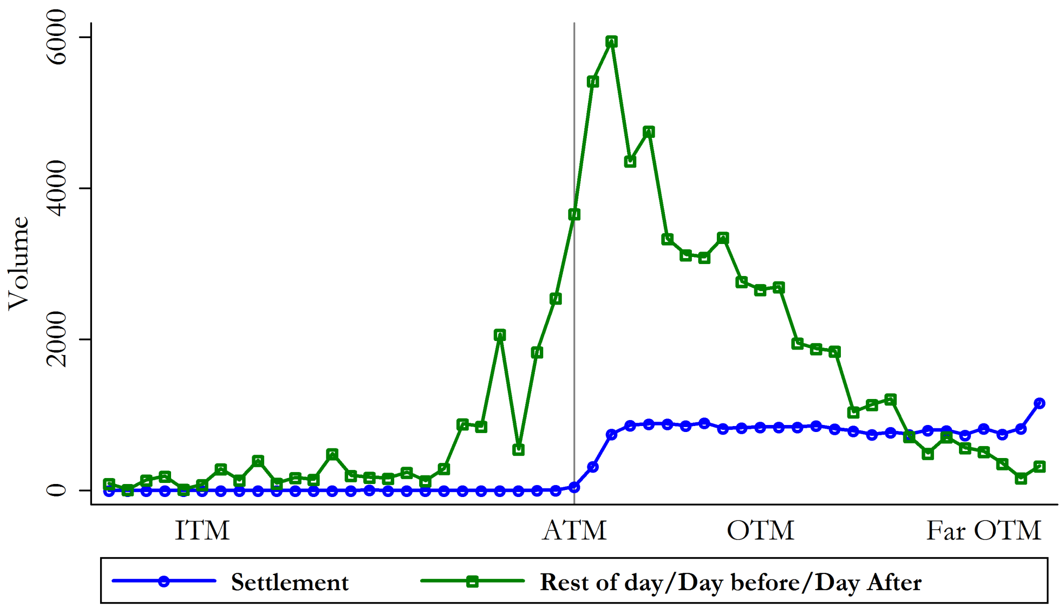
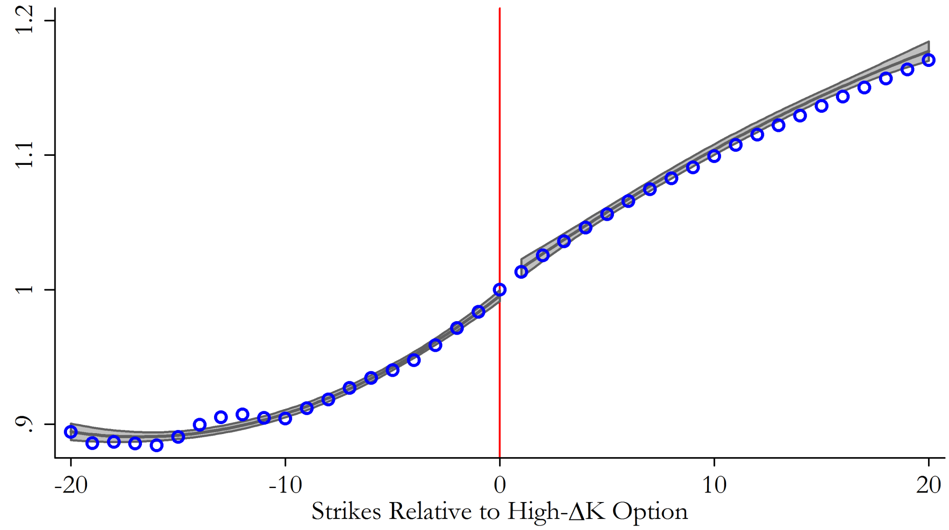


Figure IA.7. VIX Sensitivity and Trade Volume for Call Options. This figure shows the relationship between trade volume of monthly S&P 500 call options and the sensitivity of VIX settlement value to price movements in those options. The sample consists of call options used in VIX settlement calculations from January 2008 to April 2015. VIX sensitivity for each option is calculated as the change in VIX settlement value as a result of five-cent price movements in that option, holding all other prices constant. Each month's call options used in settlement calculation are divided into 25 bins based on their moneyness, and VIX sensitivity and trade volume are averaged for each bin, and then over time. Panel A shows average VIX sensitivity for each bin and Panel B shows average volume at VIX settlement. Panel C compares the settlement volume with average volume over the rest of the day, the day before, and the day after.

Panel C: Call Options Volume at Settlement versus Daily Volume on Other Days



Panel A: Changes in Strike Price



Panel B: Changes in Moneyness

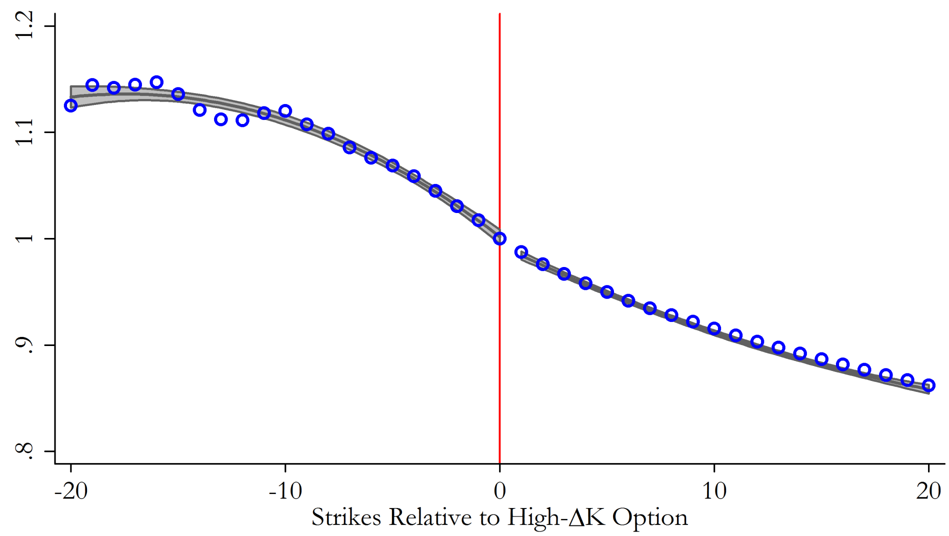
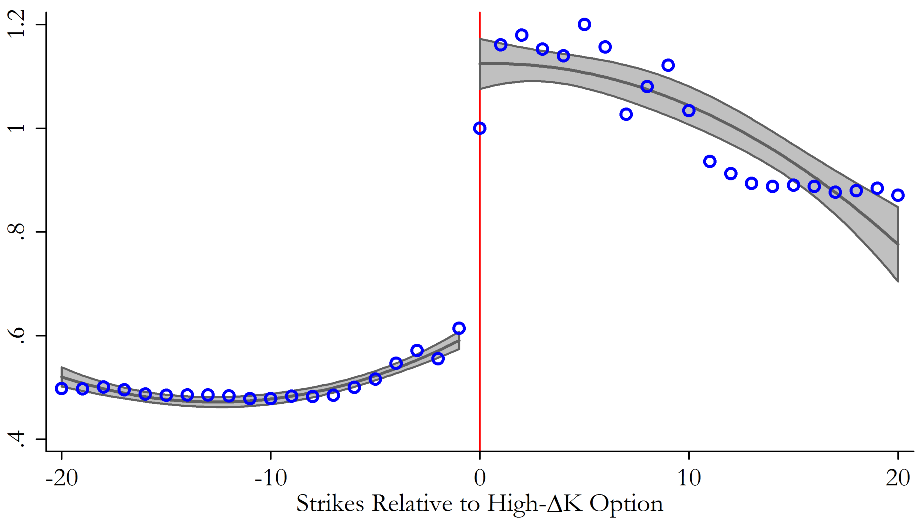


Figure IA.8. Continuity of Strike Price and Moneyness around VIX Sensitivity Jumps for Put Options. This figure shows the continuity in VIX Strike Price and Moneyness for put options around the jump in VIX sensitivity at the settlement. The sample consists of put options used in VIX settlement calculations from January 2008 to April 2015. The center vertical lines represent the strike with high ΔK at the jump. Strike prices above and below the jumps are ranked relative to strikes at the jump, and then strike price and moneyness are normalized by the values of these variables for the High-ΔK strike at the jump. The blue circles show the average of these variables for each rank across different settlements. Panels A and B show changes in strike price and moneyness around the jump. The gray areas represent the 95% confidence interval for the fitted values of the variable on the y-axis as a quadratic function of the rank of options relative to the High-ΔK option.

Panel A: VIX Sensitivity



Panel B: Settlement Volume

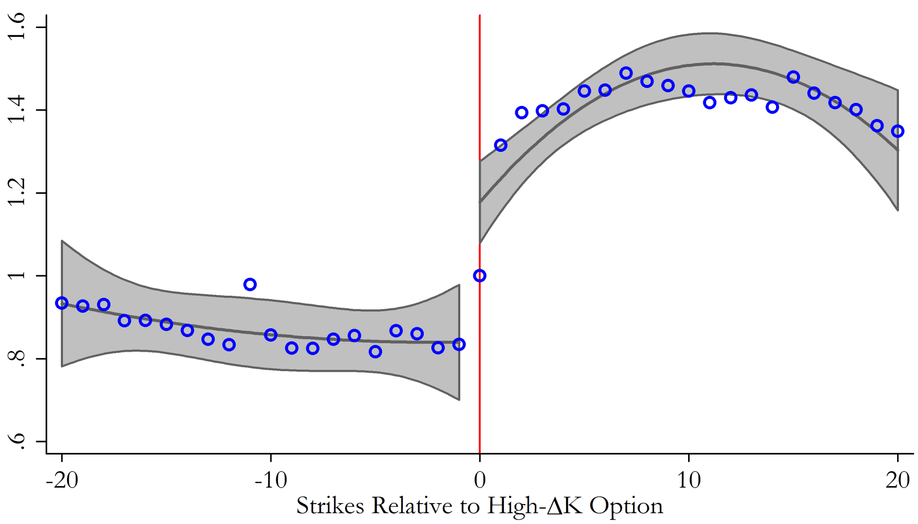
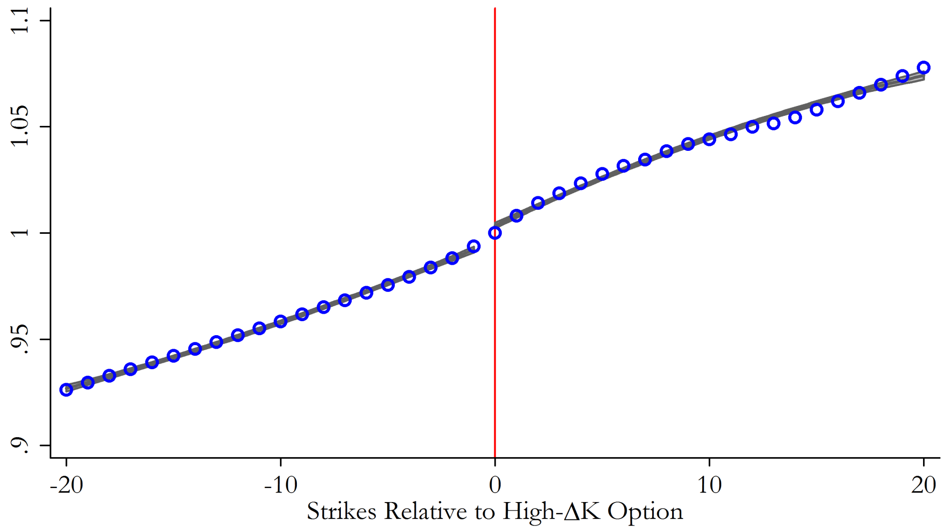
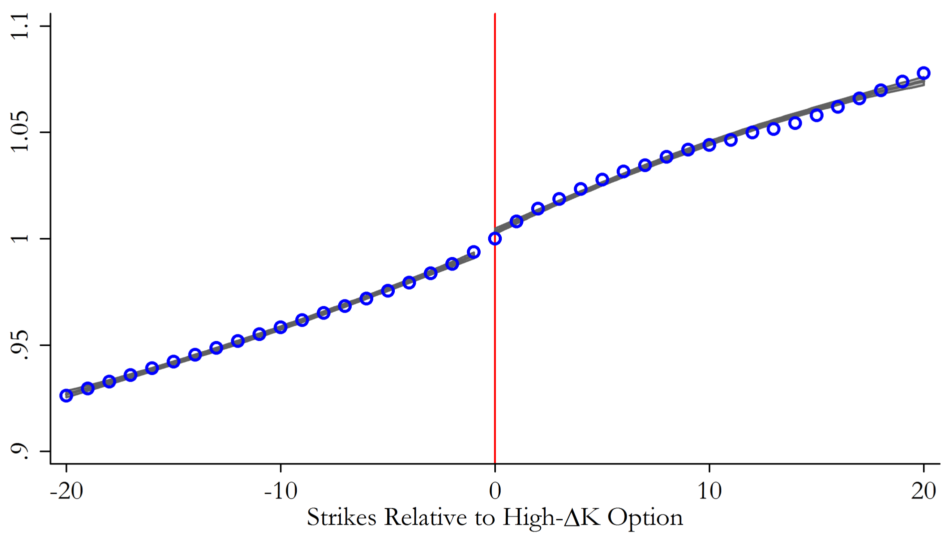


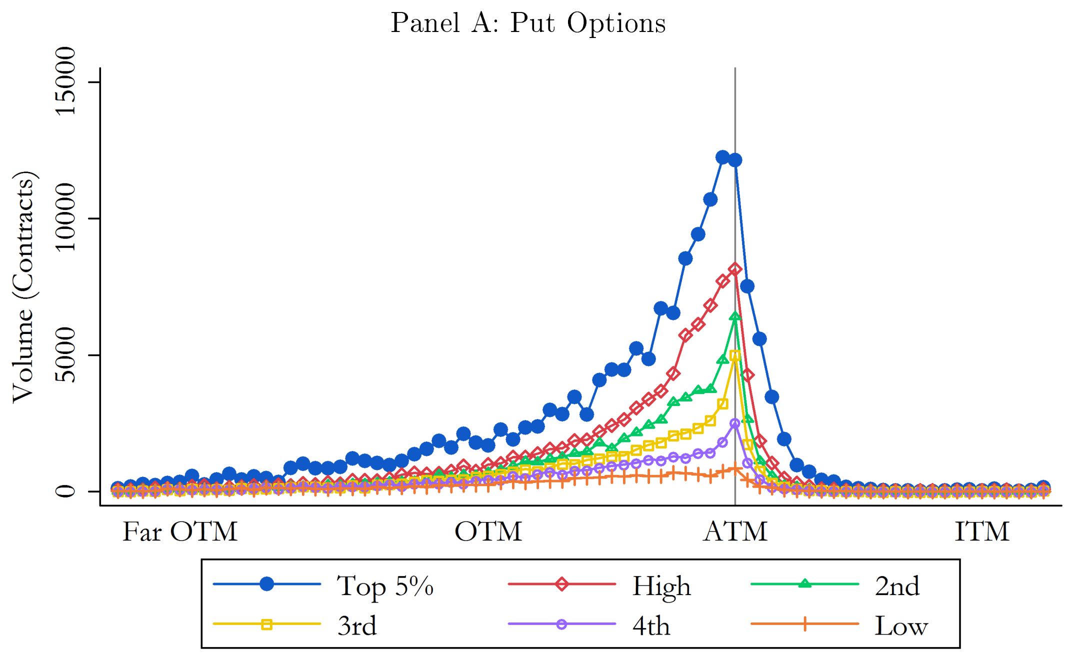
Figure IA.9. Discontinuity in VIX Sensitivity and Trade Volume of Call Options. This figure shows the discontinuity in VIX sensitivity and trade volume of call options due to the jump in strike prices. The sample consists of the options used in VIX settlement calculations from January 2008 to April 2015. A jump is defined as when ΔK (the average distance between a strike price and strike prices above and below) increases for an option relative to the adjacent strike. The center vertical lines represent the strike with high ΔK at the jump. The strike prices above and below the jumps are ranked relative to the strikes at the jump. Afterward, VIX sensitivity, settlement volume, strike price, and moneyness are normalized by the values of these variables for the High-ΔK strike at the jump. The blue circles show the averages of these variables for each rank across different settlements. Panel A shows the jump in VIX sensitivity and Panel B shows the jump in trade volume at settlement. Panels C and D show changes in strike price and moneyness around the jump. The gray areas represent the 95% confidence interval for the fitted values of the variable on the y-axis as a quadratic function of the rank of options relative to the High-ΔK option.

Panel C: Changes in Strike Price



Panel D: Changes in Moneyness





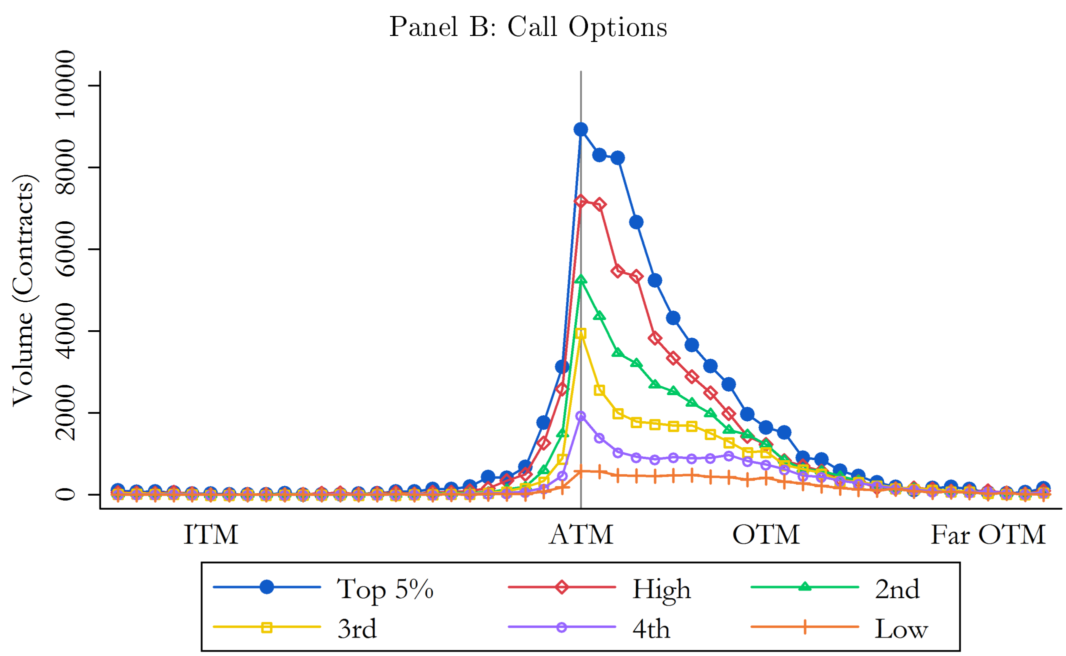


Figure IA.10. Volume in Non-Settlement Days by Volume Quantiles. This figure shows the average daily trade volume of monthly SPX options divided into quantiles of aggregate daily volume. The sample consists of the non-settlement days from January 2008 to August 2014. Non-settlement days are divided into five quantiles by their aggregate daily volume. Moreover, the top 20% quantile is divided into the top 5% and the next 15% (the high quantile). Panel A shows the daily volume for put options and Panel B for calls.

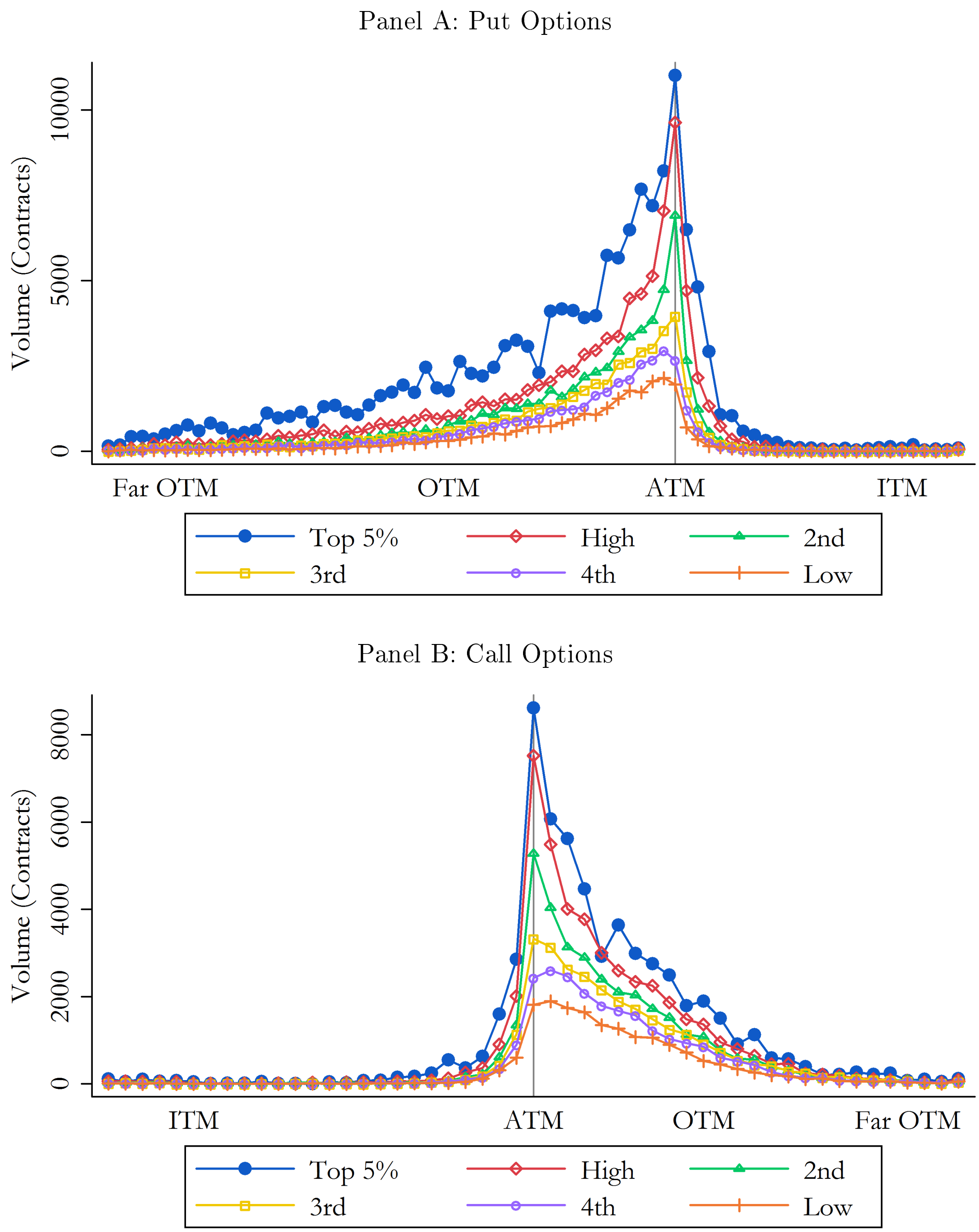


Figure IA.11. Volume in Non-Settlement Days by Alternative Volume Quantiles. This figure shows the average daily trade volume of monthly SPX options divided into quantiles of aggregate daily volume. The sample consists of the non-settlement days from January 2008 to August 2014. Non-settlement days are divided into bins by time to maturity, and then demeaned by the average volume for each time to maturity bin. Then they are divided into five quantiles by their demeaned volume. Moreover, the top 20% quantile is divided into the top 5% and the next 15% (the high quantile). Panel A shows the daily volume for put options and Panel B for calls.

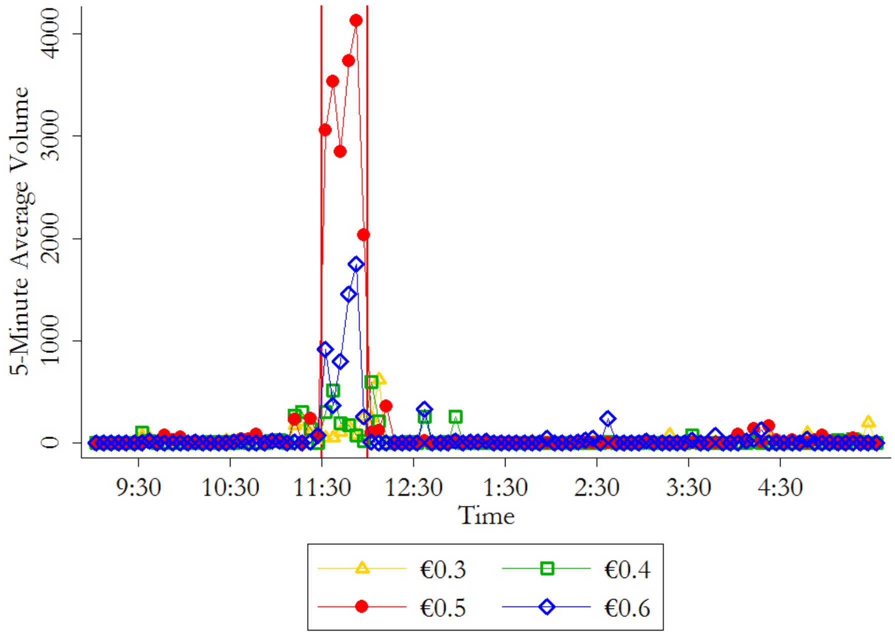


Figure IA.12. Trading Volume of EURO STOXX 50 Options around the €0.5 Cutoff. This figure shows the trade volume at different prices for EURO STOXX 50 options around the €0.5 cutoff during the VSTOXX settlement days. The trade volume at each price is aggregated over five-minute intervals, and then it is averaged across different settlement dates. The sample consists of the settlement days from January 2014 to August 2015.

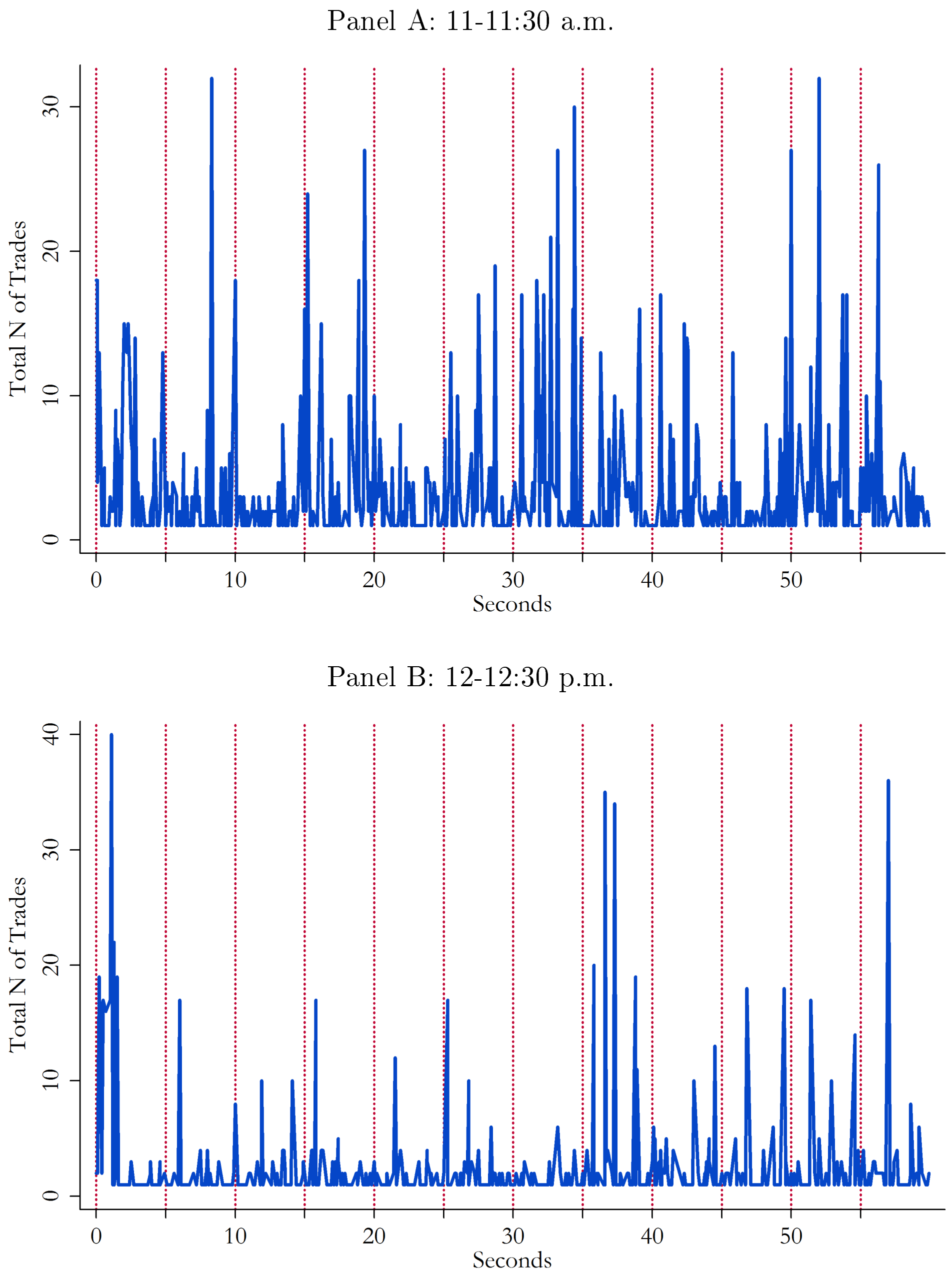


Figure IA.13. Trading Pattern of EURO STOXX 50 Put Options Outside the VSTOXX Settlement. This figure shows the number of trades over time for EURO STOXX 50 put options before and after the VSTOXX settlement. Panel A reports the trades from 11-11:30 am and Panel B from 12-12:30 pm on settlement days. Each five-second interval is divided into fifty buckets of 100 milliseconds each. The first five minutes of each 30-minute window is used to identify the bucket with the highest number of trades, and the time of that bucket is used to set the clock to zero. The graph reports the average number of trades for the 100-millisecond buckets across each minute of the next 25 minutes of the 30-minute period. The sample consists of the settlement days from January 2014 to August 2015.

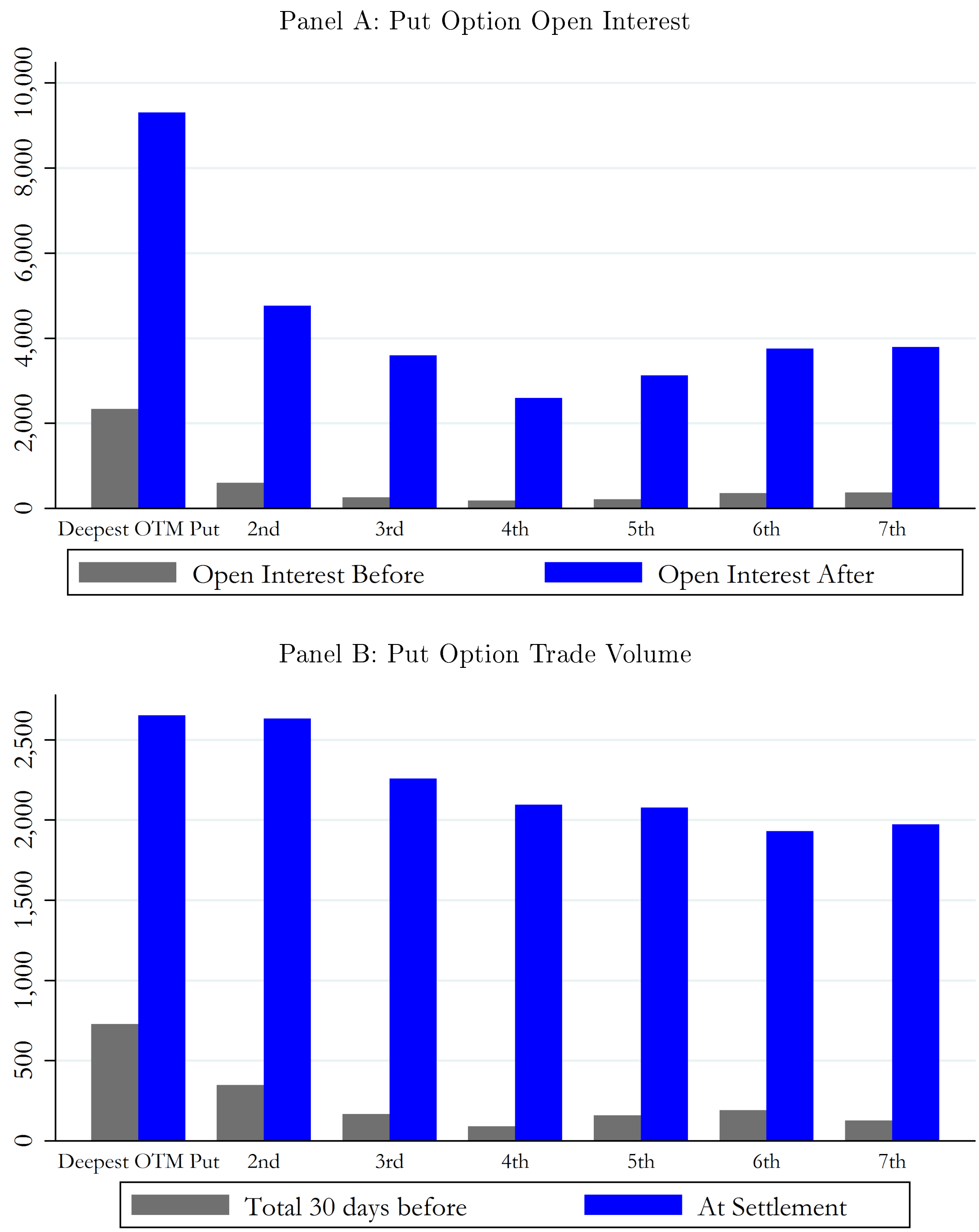


Figure IA.14. Comparison of Settlement Volume with Previous Volume and Open Interest for Put Options. This figure compares the open interest before and after the settlement for the deepest OTM put options as well as the settlement volume with the sum of the prior 30 days' trade volume. The sample consists of the put options used in VIX settlement calculations from January 2008 to August 2014. Deep OTM put options included in settlement are ranked based on their moneyness, and then their median 30-day volume and open interests are calculated across different settlement days. Panel A shows the open interest the day after settlement (blue bars) versus the day before (gray bars). Panel B shows the settlement volume (blue bars) versus total trade volume over the 30 days prior to settlement (gray bars).

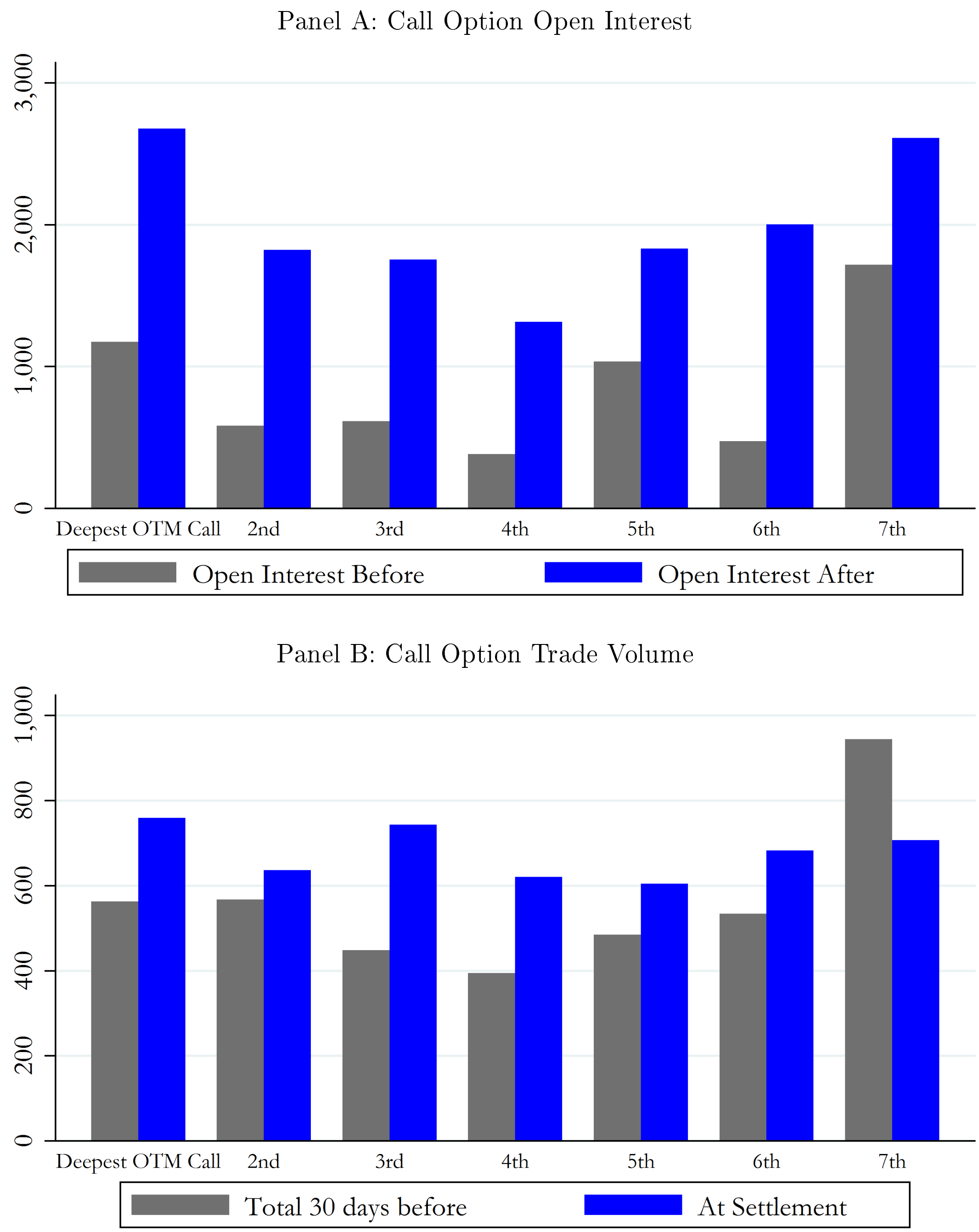


Figure IA.15. Comparison of Settlement Volume with Previous Volume and Open Interest for Call Options. This figure compares the open interest before and after settlement for the deepest OTM call options as well as the settlement volume with the sum of the prior 30 days' trade volume. The sample consists of the call options used in VIX settlement calculations from January 2008 to August 2014. Deep OTM call options included in settlement are ranked based on their moneyness, and then their median 30-day volumes and open interests are calculated across different settlement days. Panel A shows the open interest the day after the settlement (blue bars) versus the day before (gray bars). Panel B shows the settlement volume (blue bars) versus total trade volume over the 30 days prior to settlement (gray bars).

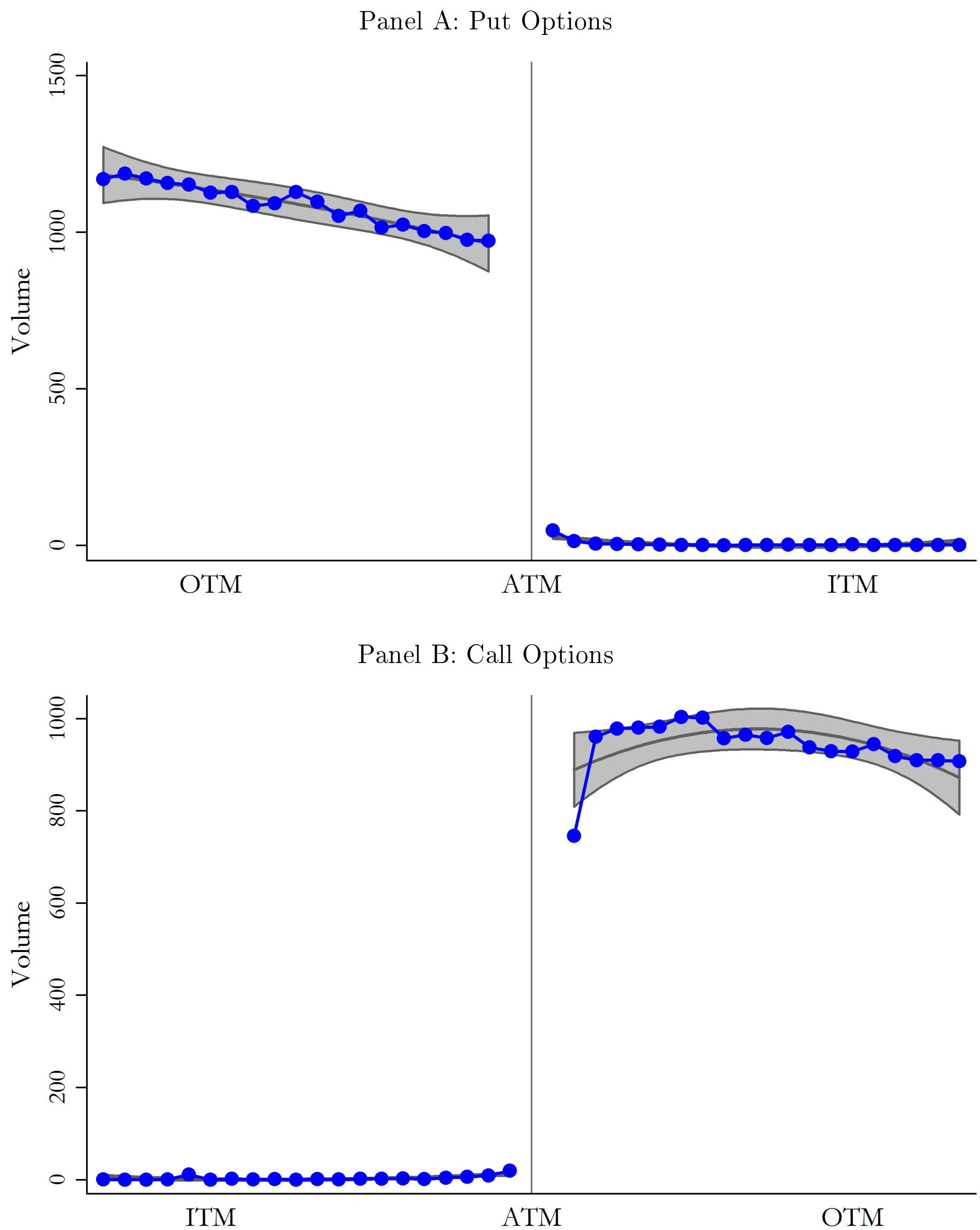


Figure IA.16. Discontinuity Around the ATM Options. This figure shows the discontinuity in trade volume at settlement right around the at-the-money threshold. The sample consists of S&P 500 monthly put options from January 2008 to April 2015. At each settlement time, options are sorted based on their moneyness, then their trade volume in the first 30 seconds of the day is averaged across all the settlement days for each rank. The strike prices right above and right below the at-the-money threshold are excluded. The gray area represents the 95% confidence interval.

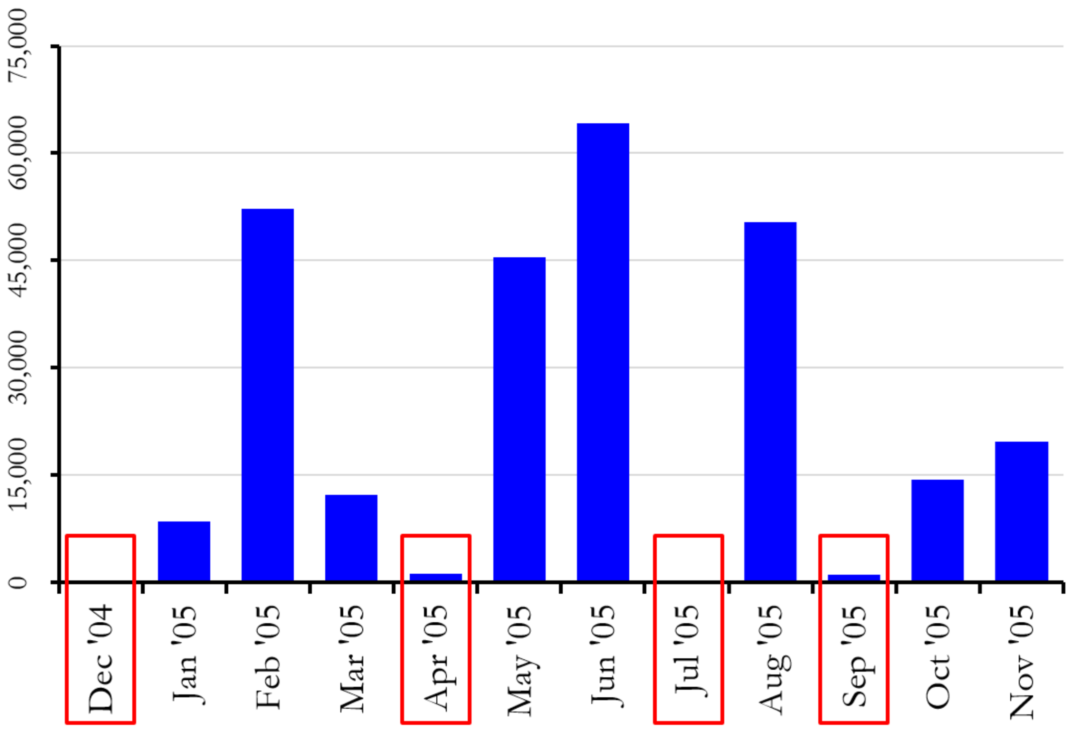


Figure IA.17. Comparison of Volume at Usual Settlement Time between Months with and without VIX Futures. This figure shows the aggregate trade volume of out-of-the-money put options in the first five minutes of the usual settlement days for months with and without VIX derivatives. The sample consists of put options used in VIX settlement calculations from December 2004 to November 2005. The trade volume is aggregated across all the strike prices of out-of-the-money put options. Months when VIX futures are not traded are circled in red. VIX futures are traded in all other months. There is no VIX option contract traded in this period.

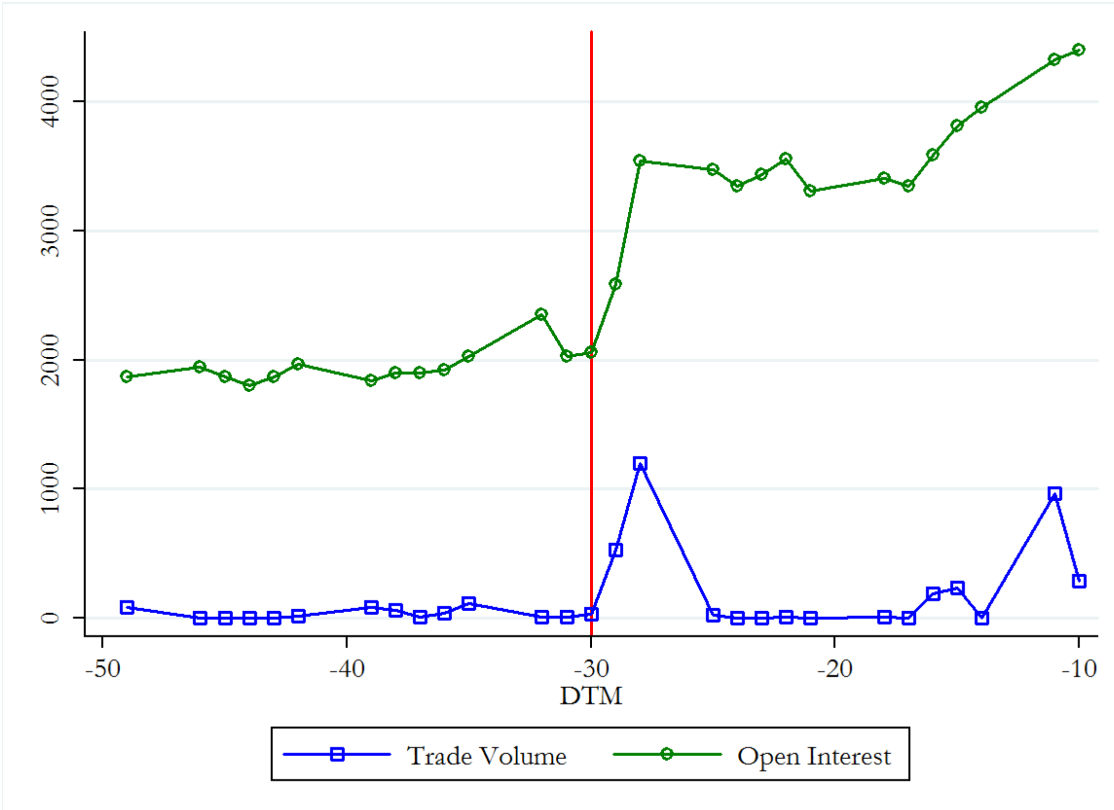


Figure IA.18. Daily Trade Volume and Open Interest of Exchange Traded Variance Swaps. This figure graphs the end of the day trade volume and open interest of CBOE exchange traded S&P 500 Variance Futures over days to maturity of these contracts. The sample consists of variance future contracts from December 2012 to August 2015. The numbers are averaged by days to maturity across series with different expiration dates. The blue squares show the trade volume and the green circles show the open interest.

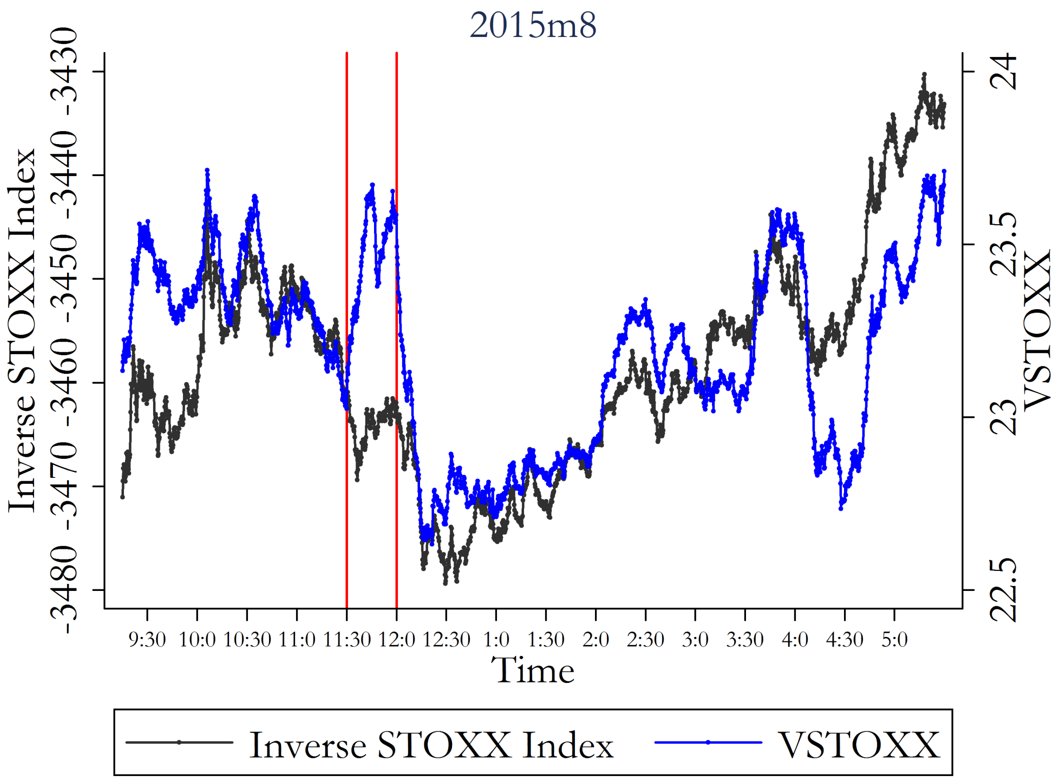


Figure IA.19. VSTOXX versus Inverse EURO STOXX 50 Index - August 19th 2015. This figure graphs the VSTOXX index and inverse of the EURO STOXX 50 index for a recent settlement day on August 19th, 2015. The blue dots show the VSTOXX values and the black dots show the inverse Euro STOXX 50 index. The time between the two vertical red lines shows the settlement calculation period.

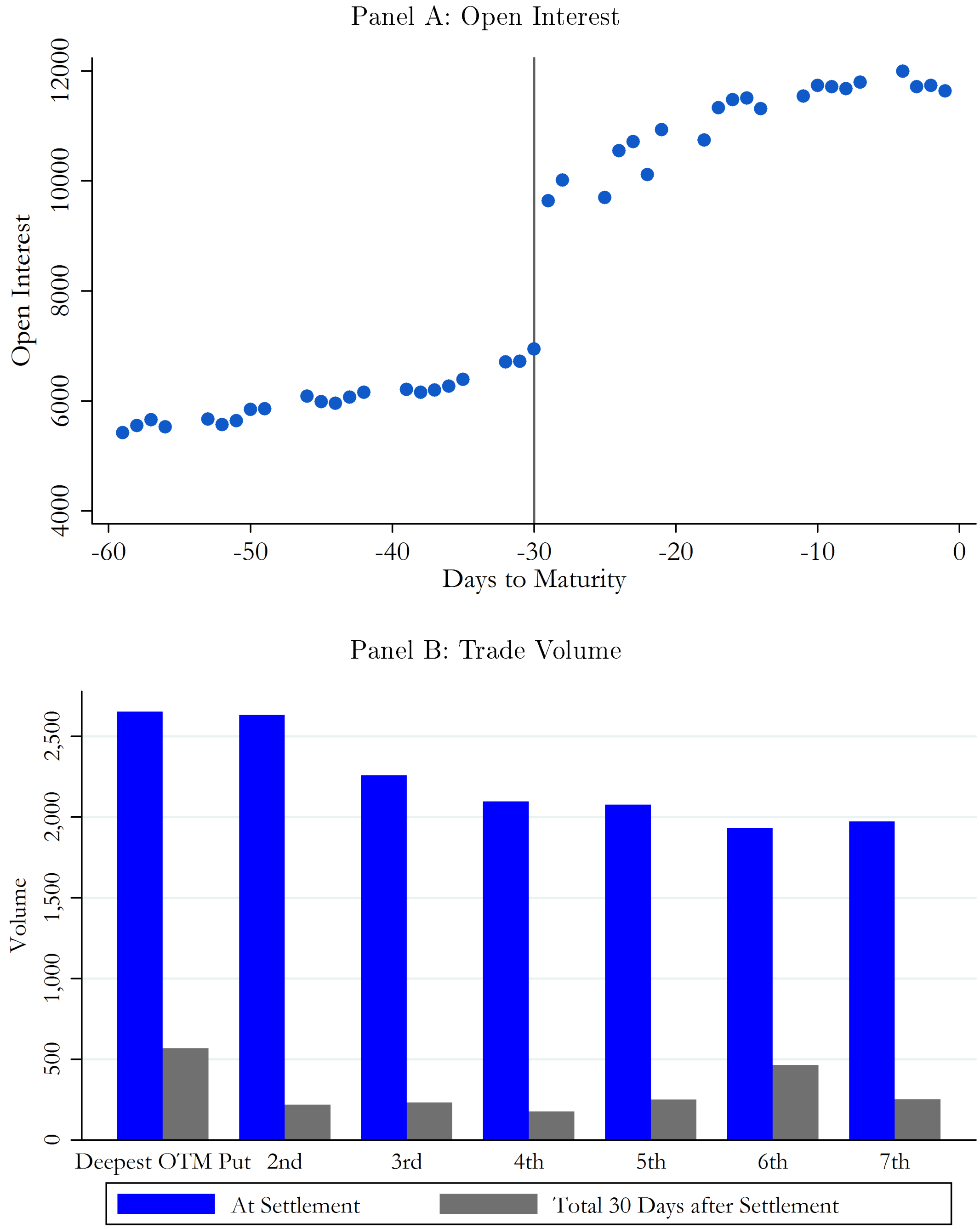


Figure IA.20. Comparison of Settlement Volume with Volume and Open Interest after Settlement for Put Options. Panel A shows the changes in open interest for the 20 deepest OTM put SPX options included in each settlement by their days to maturity. The sample consists of put options used in VIX settlement calculations from January 2008 to August 2014. Panel B shows the settlement volume (blue bars) versus total trade volume after settlement (gray bars). Deep OTM put options included in settlement are ranked based on their moneyness, and then their median 30-day volume after settlement (until maturity) are calculated across different settlement days.

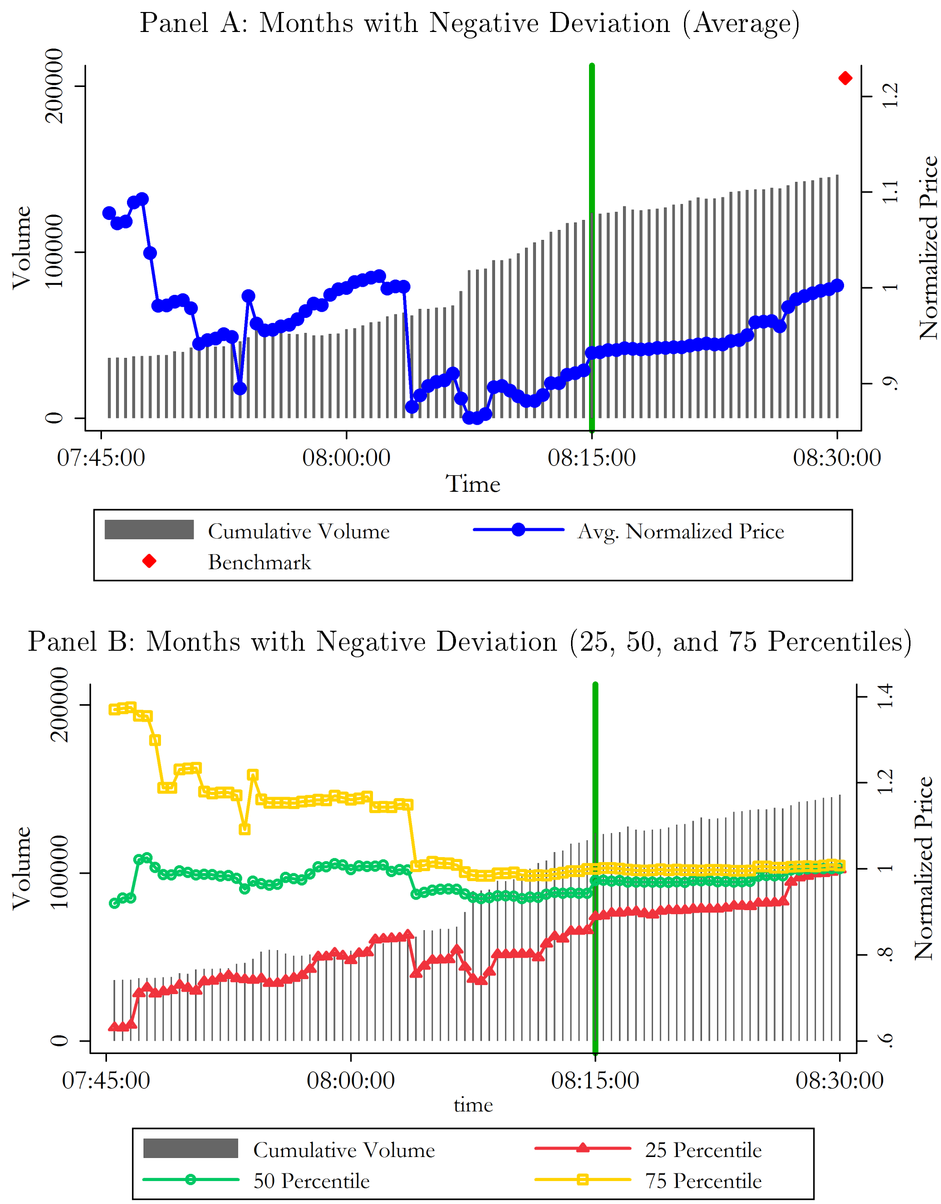
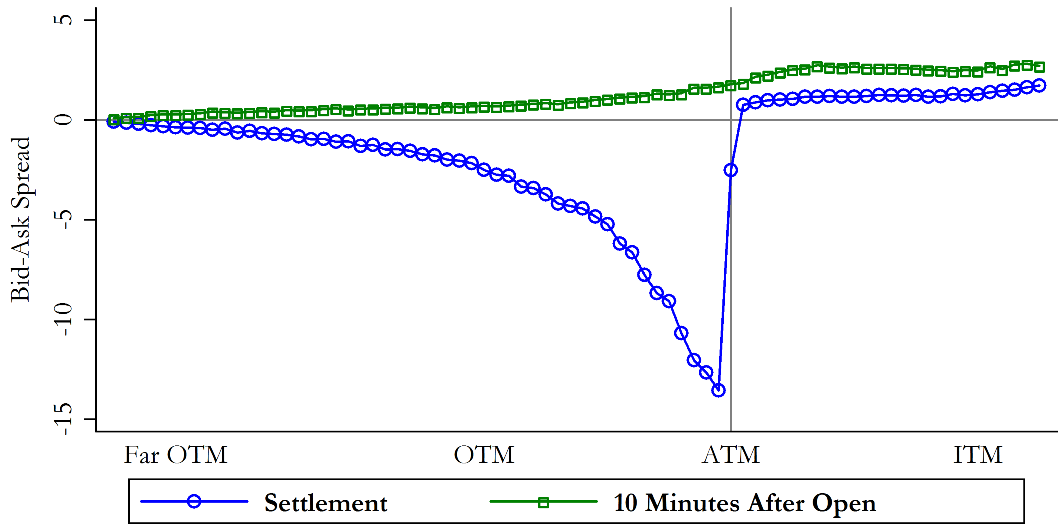


Figure IA.21. Price Movements of Put SPX Options before Market Open for Days with Negative Deviations. This figures show the change in indicative price of put SPX options over time on settlement days with negative deviations, as reported in CBOE Imbalance Reports. Prices of all put options used in settlement are normalized by their settlement price and then averaged across all strike prices and over settlement dates. Panel A shows the averages and Panel B the 25, 50, and 75 percentiles for days with negative VIX settlement deviation of less than -20bp. (March 2013 and October 2014 are removed from the sample because SPX option series do not have quotes in the data immediately after market open.)

Panel A: Bid-Ask Spread



Panel A: Percentage Bid-Ask Spread Relative to Mid-Point

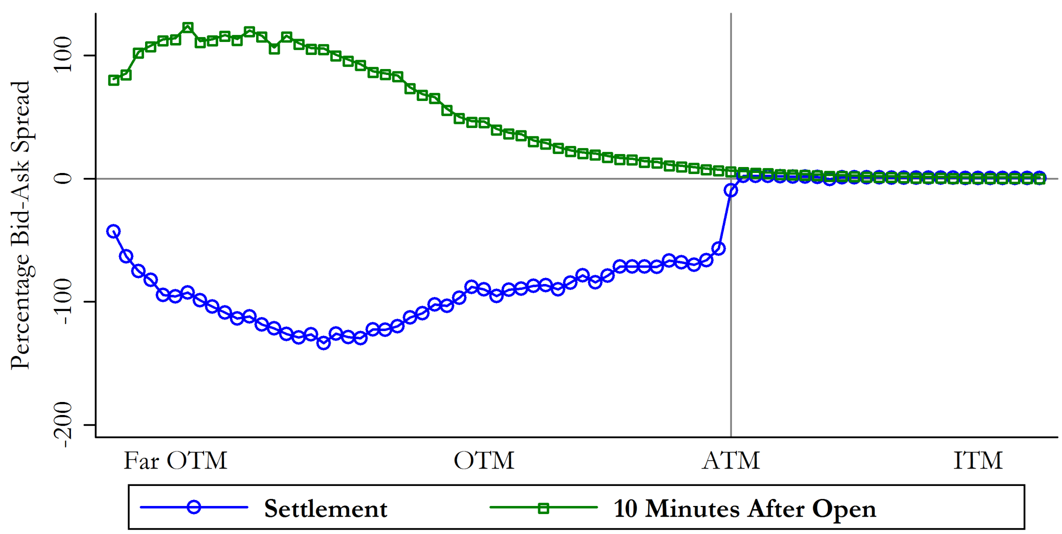


Figure IA.22. Negative Bid-Ask Spread at Settlement for Put Options. This figures compares the bid-ask spread at settlement time with the bid-ask spread 10 minutes after the market opens. The bid-ask spread at settlement is calculated using the last observed quote for each SPX options in the settlement imbalance reports, provided that the option has price quote after 8:15 am and has bid and ask size of at least 50 contracts. The sample consists of put options used in VIX settlement series from May 2010 to April 2015. The spread is aggregated over time and in different bins of moneyness. The blue circles show the spread at settlement and the green squares show it 10 minutes after open. Panel A shows the raw spread and Panel B shows the spread divided by the mid-price.

Table IA.1. Disentangling the Effect of Moneyness and ΔK. This table shows the relationship between trade volume and VIX sensitivity of monthly SPX options and their moneyness and ΔK measure in a regression of the form:

$$Y_{it} = \beta_0 + \beta_1 OTMness_{it} + \beta_2 \Delta K_{it} + \alpha_t + \epsilon_{it}$$

where Y_{it} is the VIX sensitivity of strike i at time t in Panel A and the settlement volume in Panel B, $OTMness_{it}$ is the percentage that the strike is out-of-the money relative to ATM strikes, ΔK is the average distance from the strike prices above and below, and α_t is the date fixed effect. The sample consists of the option chains used in the settlement dates from January 2008 to April 2015. Reported t-statistics in parentheses are clustered by date.

Panel A: VIX Sensitivity

	Put_Sensitivity	Call_Sensitivity	Put_Sensitivity	Call_Sensitivity
OTMness	0.789***	0.0826	0.368***	-0.114***
	(6.44)	(1.90)	(7.35)	(-9.26)
Delta K			0.0525***	0.0129***
			(6.38)	(24.54)
Constant	-0.746***	-0.0200	-0.687***	0.114***
	(-5.43)	(-0.43)	(-10.08)	(9.04)
Date FE	Yes	Yes	Yes	Yes
Observations	7,097	3,129	7,097	3,129
Adjusted R^2	0.419	0.326	0.776	0.967

Panel B: Settlement Volume

	Put_Volume	Call_Volume	Put_Volume	Call_Volume
OTMness	5561.1***	139.2	2544.1***	-1324.5***
	(5.16)	(0.38)	(5.65)	(-6.31)
Delta K			376.9***	96.09***
			(5.40)	(7.96)
Constant	-4066.8**	761.6	-3641.3***	1758.7***
	(-3.36)	(1.96)	(-4.00)	(8.25)
Date FE	Yes	Yes	Yes	Yes
Observations	7,097	3,129	7,097	3,129
Adjusted R^2	0.392	0.584	0.605	0.702

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table IA.2. Discontinuity in VIX Sensitivity and Trade Volume. This table shows the discontinuity in VIX sensitivity and trade volume due to the jump in strike prices. The sample consists of the options used in settlement calculations from January 2008 to April 2015. A jump is defined as when the ΔK (the average distance between a strike price and the strike prices above and below) increases for an option relative to the adjacent strike price. For each jump, the two observations right around the jump are included in the two groups of HighΔK and LowΔK. Panel A compares the characteristics of those options. Panel B shows a 2SLS instrumental variable regression of volume on VIX sensitivity in a regression of the form:

$$Volume_{it} = \beta_0 + \beta_1 VIX\hat{Sensitivity}_{it} + \alpha_t + \epsilon_{it}$$

where in the first stage the jump is used as an instrument for the volume in the following regression:

$$VIX\hat{Sensitivity}_{it} = \pi_0 + \pi_1 Jump_{it} + \alpha_t + \varepsilon_{it}$$

α_t is the date fixed effect. Reported t-statistics in parentheses are clustered by settlement date.

Panel A: Characteristics at the Jump

	Put Options				Call Options			
	LowΔK	HighΔK	Difference	t-stats	LowΔK	HighΔK	Difference	t-stats
Strike	971.97	965.90	-6.07	(-0.29)	1479.11	1484.50	5.39	(0.18)
OTMness	36.35	37.80	1.45	(0.73)	10.14	10.53	0.39	(0.48)
VIX Sensitivity	0.28	0.44	0.16***	(4.46)	0.09	0.14	0.05***	(8.19)
Settle Vol.	2335.01	3329.63	994.62***	(3.45)	847.17	1276.79	429.62***	(4.47)
Non-Settle Vol.	2252.73	2193.82	-58.91	(-0.20)	1677.07	1960.71	283.64	(0.87)
Observations	966				560			

Panel B: The Jump as an Instrument

	At Settlement		At Other Times		
	Settle Vol.	Settle Vol.	Day Before	Rest of the Day	Day After
VIX Sensitivity	6473.5***	6473.5***	770.8	1235.7	1283.3
	(7.56)	(7.56)	(0.41)	(0.60)	(0.52)
Constant	438.9***	1153.3***	1454.1***	2079.5***	4468.5***
	(4.73)	(10.03)	(5.82)	(7.51)	(13.51)
Date FE	No	Yes	Yes	Yes	Yes
Observations	1,526	1,526	1,389	1,523	1,429
Adjusted R^2	0.367	0.545	0.026	0.053	0.056

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table IA.3. Relationship between Moneyness and Pre-Settlement Liquidity. This table shows the relationship between the moneyness of OTM put and call SPX options and their liquidity in pre-settlement times in a regression of the form:

$$Liquidity_{it} = \beta_0 + \beta_1 Moneyness_{it} + \alpha_t + \epsilon_{it}$$

where $Moneyness_{it}$ is the percentage that the strike is close to the money relative to ATM strikes, $Liquidity_{it}$ is measured as either the average of prior 30 days volume or the average of prior 30 days inverse percentage bid-ask spread, and α_t is the date fixed effect. The sample consists of the option chains used in the settlement dates from January 2008 to April 2015. Reported t-statistics in parentheses are clustered by date.

	Liquidity (Volume)		Liquidity (Inv. Pct. Spread)	
	Calls	Puts	Calls	Puts
Moneyness	11698.0***	6287.6***	1.002***	0.377***
	(7.61)	(18.56)	(12.11)	(20.83)
Constant	-10072.0***	-4872.2***	-0.724***	-0.273***
	(-6.95)	(-16.04)	(-9.28)	(-16.84)
Date FE	Yes	Yes	Yes	Yes
Observations	3129	7097	3129	7097
Adjusted R^2	0.122	0.124	0.736	0.685

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table IA.4. Volume and Open Interest in Months with and without VIX Futures. This table shows an OLS regression of trade volume and open interest at usual settlement days in the following regression:

$$Y_{it} = \beta_0 + \beta_1 WithFutures_t + \epsilon_{it}$$

where Y_{it} is the trade volume of strike i at time t in Panel A and the open interest in Panel B, $WithFutures_t$ is a dummy variable that takes the value of one for the months when VIX future contracts were traded and zero otherwise. The sample consists of out-of-the-money put options used in VIX settlement calculations from December 2004 to November 2005. There is no VIX option contract traded in this period. The dependent variable in Panel A is trade volume at the first five minutes of usual settlement days. The dependent variable in Panel B is the open interest the day before settlement. Reported t-statistics in parentheses are clustered by date.

Panel A: Trade Volume at First Five Minutes of Settlement Day

	All Options	OTM Options	ITM Options
With Futures	295.1**	593.6**	1.191
	(4.37)	(4.31)	(0.72)
Constant	6.893*	12.07*	1.778
	(2.70)	(2.49)	(1.95)
Observations	1,345	618	618
Adjusted R^2	0.082	0.191	-0.001

Panel B: Open Interest before the Settlement

	All Options	OTM Options	ITM Options
With Futures	2001.9	364.4	1762.4
	(0.71)	(0.08)	(1.47)
Constant	9547.8***	16901.5***	2352.7***
	(6.67)	(5.89)	(6.45)
Observations	1,345	618	618
Adjusted R^2	0.001	-0.002	0.005

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table IA.5. Put SPX Options Settlement Prices Relative to the Spread at Benchmark. This table shows the results of a paired t-test of the difference between put SPX option prices at settlement and the bid, ask, and mid-quote of the options at the market open immediately after the settlement (Panels A) and at the close the day before settlement (Panels B). Positive deviation days are those with 20bp or greater deviation in VIX at settlement relative to the open benchmark, and negative deviation days are those with -20bp or less deviation. The sample consists of the settlement days from May 2010 to April 2015. For options with either missing quotes or quotes that are strictly inferior to other strikes in the same option series, we use quotes of options that strictly dominate that option, i.e. the best ask of the more ITM options or the best bid of the more OTM options. The standard errors are clustered by date and bins of moneyness.

Panel A: Relative to the Quotes at the Open

	Pos. VIX Deviation Rel. to Open			Neg. VIX Deviation Rel. to Open		
	RelToBid	RelToMid	RelToAsk	RelToBid	RelToMid	RelToAsk
	0.379***	0.177**	-0.0219	-0.0198	-0.180*	-0.341**
	(4.11)	(3.25)	(-1.13)	(-0.36)	(-2.03)	(-2.58)
Observations	2155	2155	2155	910	910	910

Panel B: Relative to the Quotes at Previous Close

	Pos. VIX Deviation Rel. to Open			Neg. VIX Deviation Rel. to Open		
	RelToBid	RelToMid	RelToAsk	RelToBid	RelToMid	RelToAsk
	0.715**	0.344**	-0.0259	0.0662	-0.224*	-0.515*
	(3.25)	(3.27)	(-0.64)	(1.17)	(-2.14)	(-2.41)
Observations	2155	2155	2155	910	910	910

Table IA.6. Reversal of Movement in SPX Options Prices around the Settlements. This table shows the relationship between changes in the prices of individual SPX options from the previous close to the settlement and from the settlement to the market open in a regression of the form:

$$Price\_Ch\_SettleToOpen_{it} = \beta_0 + \beta_1 Price\_Ch\_CloseToSettle_{it} + \alpha_t + \epsilon_{it}$$

where *Price_Ch_SettleToOpen* is the change of SPX option prices from settlement to the mid-quote at the market open and *Price_Ch_CloseToSettle* is the change from the mid-quote of the previous close to the settlement price. The sample consists of the option chains used in the settlement dates from May 2010 to April 2015. Reported t-statistics in parentheses are clustered by date and bins of moneyness.

	PriceChange_SettlementToOpen			
	Puts	Puts	Calls	Calls
PriceChange_Prev.CloseToSettlement	-0.581***	-0.578***	-0.473***	-0.527***
	(-6.58)	(-6.07)	(-5.81)	(-5.25)
Constant	0.0211	0.0106	-0.00134	0.00952
	(1.07)	(0.55)	(-0.07)	(0.27)
ExpirationDate FE	No	Yes	No	Yes
Observations	3483	3483	1240	1240
Adjusted R^2	0.74	0.83	0.57	0.74

Table IA.7. Comparison of the VIX Settlement Deviations Using Different Benchmarks. This table shows the relationship between different benchmarks used to gauge the magnitude of VIX deviations at settlement. Panel A shows the correlation between deviations of the settlement from multiple benchmarks for settlement dates from May 2010 to April 2015. The benchmark at open is the VIX index calculated using mid-quotes of SPX options at open. The previous close benchmark is the VIX index calculated using mid-quotes at the close of the previous day, adjusted by adding price changes of the second-term VIX futures from close to open to it. All the benchmarks are calculated using the same range of SPX options included in both settlement and the open benchmark. The ITM benchmark is calculated using the put-call parity implied value of the OTM options from the mid-quote of the ITM options reported in the settlement imbalance reports. PrevClose_BS_Adjusted is the VIX calculated using the mid-quotes at previous close, adjusted for changes in the market index, time decay, and overnight changes in volatility, as explained in Appendix IA.B. Panel B reports the results of a paired t-test between deviations relative to open and previous close benchmarks for both days with positive and negative deviations from January 2008 to April 2015.

Panel A: Correlation of Deviations Using Different Benchmarks

	Open	PrevClose	ITM	PrevClose_BS_Adjusted
Open	1			
PrevClose	0.747***	1		
ITM	0.737***	0.653***	1	
PrevClose_BS_Adjusted	0.785***	0.779***	0.662***	1

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel B: Deviation from Open versus Previous Close

	Pos. Deviation from Open				Neg. Deviation from Open			
	Open	PrevClose	Difference	t-stats	Open	PrevClose	Difference	t-stats
Deviation	0.28	0.63	0.35*	(2.16)	-0.39	-0.81	-0.42**	(-3.14)

Table IA.8. Deviation Contribution by Percentage and per Dollar Amount for Different Groups of Options. This table reports the source of deviation across different groups of options. Panel A measures the deviations using the open benchmark, and panel B using the Black-Scholes adjusted previous close benchmark. The sample consists of option chains used in settlement dates from May 2010 to April 2015. The first two rows show the percentage contribution of each group of options to the aggregate VIX deviations for days with large positive and negative deviations. At each settlement the sample of put options included in the settlement and the open benchmark is divided into three subgroups with equal number of strikes based on their moneyness: 1) Close-in OTM Puts, 2) Deeper OTM Puts, and 3) Very Deep OTM Puts. Call options are examined as a separate group. The VIX deviation caused by each group is calculated as the deviation caused by all individual options in that subgroup settling at the actual settlement price, while holding the prices of options in other groups the same as the benchmark. The deviations caused by each group is averaged across settlement dates, and the percentage of deviation caused by each group is reported here. Deviations per dollar is the deviation caused by each group of options divided by the aggregate dollar value overspent on that group of option relative to the mid-quote of the benchmark:

$$\frac{bpDeviations}{\sum_i 100 * Volume_i * (Price_{i,Settlement} - Price_{i,Benchmark})}$$

where the numerator shows total basis point deviation caused by each group of option and the denominator shows total money overspent for all the strikes i in each group. The average of this measure across settlements is shown in the fourth and fifth rows of the table.

Panel A: Deviations Relative to the Open Benchmark

Total Deviations	OTM Puts	Deep OTM Puts	Very Deep OTM Puts	Calls
Positive	28.64	21.53	21.53	28.29
Negative	43.64	23.9	9.87	22.6
bp Dev per $1M Overpaid:				
Positive	0.11	0.12	0.12	0.11
Negative	0.15	0.15	0.18	0.16
Avg N of Strikes	31.4	31.7	31.9	35.9

Panel B: Deviations Relative to the Previous Close Benchmark

Total Deviations	OTM Puts	Deep OTM Puts	Very Deep OTM Puts	Calls
Positive	39.56	23.38	19.17	17.89
Negative	48.74	23.02	8.51	19.73
bp Dev per $1M Overpaid:				
Positive	0.11	0.12	0.12	0.11
Negative	0.14	0.15	0.18	0.14
Avg N of Strikes	31.4	31.7	31.9	35.9

Table IA.9. Market Distortions Associated with VIX Settlement Deviations. The table below lists the distortions for VIX options, futures, and total values by year. Market distortions are calculated as the product of the open interest of affected VIX derivatives on close of the day before settlement and the magnitude of VIX deviation relative to the open benchmark. The sample consists of the settlement dates from January 2008 to April 2015.

VIX Market Distortions (Dollars in Millions)			
Year	**Futures on VIX**	**Options on VIX**	**Total**
2008	$37	$44	$81
2009	$34	$83	$117
2010	$51	$164	$215
2011	$140	$353	$493
2012	$88	$200	$288
2013	$123	$208	$331
2014	$75	$174	$249
2015	$15	$26	$41
Total	**$563**	**$1,252**	**$1,815**

Table IA.10. Market Distortions Associated with Settlement Deviations for Other Volatility Indices. The table below lists the cost distortions by year from settlement deviations on futures and options of volatility indices for crude oil, emerging markets, Russell 2000, Brazil, gold, NASDAQ, and S&P Short Term. Market distortions are calculated as the open interest of affected derivatives multiplied by the magnitude of deviation. Options data in the table sample ends in August 2014.

		Market Distortions (Dollars in Thousands)					
		2007-2011	2012	2013	2014	2015	Total
Crude Oil							
	Futures	-	$113	$17	$441	$524	$1,095
	Options	-	$4	$6	$6	-	$16
	Total	**-**	**$117**	**$23**	**$447**	**$524**	**$1,111**
Emerging Markets							
	Futures	-	$550	$284	$833	$9	$1,676
	Options	-	$6	$6	$68	-	$80
	Total	**-**	**$556**	**$290**	**$901**	**$9**	**$1,756**
Russell 2000							
	Futures	$2,928	-	$1	$346	$0	$3,275
	Options	$605	-	-	$55	-	$660
	Total	**$3,533**	**-**	**$1**	**$401**	**$0**	**$3,935**
Brazil							
	Futures	-	$12	$9	$1,006	$2	$1,029
	Options	-	-	-	-	-	$0
	Total	**-**	**$12**	**$9**	**$1,006**	**$2**	**$1,029**
Gold							
	Futures	$1	$97	$40	$71	$11	$220
	Options	$0	$1	$63	$6	-	$70
	Total	**$1**	**$98**	**$103**	**$77**	**$11**	**$290**
NASDAQ							
	Futures	$465	$4	$123	$730	$50	$1,373
	Options	$98	-	-	-	-	$98
	Total	**$563**	**$4**	**$123**	**$730**	**$50**	**$1,471**
S&P Short Term							
	Futures	-	-	-	$1,630	$118	$1,749
	Options	-	-	-	-	-	-
	Total	**$0**	**$0**	**$0**	**$1,630**	**$118**	**$1,749**

Table IA.11. OLS Regression of VIX Settlement Deviations on Trade Volume and Range of Options Included. Panel A shows OLS estimates for the following regression:

$$AbsVIXDeviation_t = \beta_0 + \beta_1 Log(SettleVol_t) + \sum \gamma_i Log(NonSettleVol_{it}) + \epsilon_{it}$$

where $AbsVIXDeviation_t$ is the absolute value of VIX deviations relative to the open benchmark calculated from the same range of individual SPX options right after the market open t, calculated as reported in Figure 8. $Log(SettleVol_t)$ is the logarithm of the trade volume of all the SPX option contracts used in VIX settlement at t. The second column controls for volume in days around the settlement. Panel B shows OLS estimates for the following specification:

$$VIXDeviation_t = \beta_0 + \beta_1 TailExtension_t + \epsilon_{it}$$

where $VIXDeviation_t$ is the VIX deviation at settlement time t. $TailExtension_t$ is calculated as the last OTM put option strike price included in the likely VIX series reported by CBOE on the close of the day before minus the last put strike price included in settlement, divided by the S&P 500 index level. The sample consists of settlement dates from January 2008 to April 2015. Days where the SPX option series used to calculate the VIX do not open right after the open are excluded. Reported t-statistics in parentheses are heteroscedasticity robust.

Panel A: Absolute VIX Settlement Deviation and Trade Volume

	Abs. VIX Deviation	Abs. VIX Deviation
Log Settlement Vol.	0.130**	0.126*
	(2.74)	(2.42)
Log Vol. Rest of Day		0.0480
		(0.43)
Log Vol. Day Before		0.0246
		(0.28)
Log Vol. Day After		-0.0497
		(-0.69)
Constant	-1.260*	-1.486
	(-2.22)	(-1.29)
Adjusted R^2	0.062	0.027

Panel B: VIX Settlement Deviation and Range of Options Included

	VIX Deviation
Tail Extension	0.0254**
	(2.84)
Constant	-0.0433
	(-0.75)
Adjusted R^2	0.066

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$